2.5.B Exponential Function Context and Data Modeling

AP Precalculus

Solutions

2.5.B Practice

Identify the percent increase or decrease of each function.

1. $f(n) = 0.2(1.9)^n$

   $1.9 = 1 + \text{inc.}$

   $0.9 = \text{inc.}$

   $\boxed{\text{90\% increase}}$

2. $f(x) = 19(0.4)^x$

   $0.4 = 1 - \text{dec.}$

   $-0.6 = -\text{dec.}$

   $\boxed{\text{60\% decrease}}$

3. $f(n) = 5.2(3.302)^n$

   $3.302 = 1 + \text{inc.}$

   $2.302 = \text{inc.}$

   $\boxed{\text{230.2\% increase}}$

4. $f(x) = 801(0.965)^x$

   $0.965 = 1 - \text{dec.}$

   $-0.035 = -\text{dec}$

   $\boxed{\text{3.5\% decrease}}$

5. $f(x) = 58(0.8)^x$

   $0.8 = 1 - \text{dec.}$

   $-0.2 = -\text{dec.}$

   $\boxed{\text{20\% decrease}}$

6. $f(n) = 10(2.061)^n$

   $2.061 = 1 + \text{inc.}$

   $1.061 = \text{inc.}$

   $\boxed{\text{106.1\% increase}}$

For each problem, create a function to model the scenario.

7. Mr. Sullivan has 24 basketballs in his garage. The number of basketballs b increases at a rate of 37.6% per year t.

   $\boxed{b(t) = 24(1.376)^t}$

8. There are 1,000 fish f in a pond that are dying off at a rate of 2.5% per month m.

   $\boxed{f(m) = 1{,}000(0.975)^m}$

9. Plutoniu-238 has a half-life of 88 years t. A scientist is studying a 3-gram sample s.

   $\boxed{s(t) = 3\left(\frac{1}{2}\right)^{t/88}}$

10. A stock portfolio has a value v of \$2,400 and doubles every 13 years t.

$\boxed{v(t) = 2400(2)^{t/13}}$

11. Mr. Brust bought a new trailer. It cost him \$35,000. Unfortunately, its value v depreciates in value by 6.5% per year t.

$\boxed{v(t) = 35{,}000(0.935)^t}$

12. A new disease is killing the local frog population. The half-life of the population p is 2 weeks w. There are currently 6,000 frogs in the swamp.

$\boxed{p(w) = 6000\left(\frac{1}{2}\right)^{w/2}}$

13. Mr. Kelly's house has 8 mice running around. Their population p increases at a 450% rate per month m.

$\boxed{p(m) = 8(5.5)^m}$

14. The number of people p living in Sullyana doubles every 130 years t. There is currently one person living there.

$\boxed{s(t) = (2)^{t/130}}$

15. A population p of fruit flies increases by 14.34% each day d. There were 20 fruit flies in the house when you left.

$\boxed{p(d) = 20(1.1434)^d}$

16. Mr. Kelly's youngest child breaks the dishes on a regular basis. If Mr. Kelly never buys new dishes, then the number of dishes d he owns has a half-life of 10 months m. He currently has 60 dishes.

$\boxed{d(m) = 60\left(\frac{1}{2}\right)^{m/10}}$

17. 900 grams of radioactive material m decays at a rate of 2.6% per year t.

$\boxed{m(t) = 900(0.974)^t}$

18. A culture of bacteria has 500 cells c that doubles every 4 hours h.

$\boxed{c(h) = 500(2)^{h/4}}$

For each of the problems below, identify how the equivalent form reveals a different property.

19. If $f(m) = 5^m$ indicates that the quantity increases by a factor of 5 every month, then what does $f(m) = (5^{12})^{(m/12)}$ indicate?

The quantity increases by a factor of 5^{12} every year.

20. If $f(d) = 1.2^d$ indicates that the quantity increases by a factor of 1.2 every day, then what does $f(d) = (1.2^7)^{(d/7)}$ indicate?

The quantity increases by a factor of 1.2^7 every week.

21. If $f(s) = 1.02^s$ indicates that the quantity increases by a factor of 1.02 every second, then what does $f(s) = (1.02^{60})^{(s/60)}$ indicate?

The quantity increases by a factor of 1.02^{60} every minute.

22. If $f(t) = 2^t$ indicates that the quantity increases by a factor of 2 every year, then what does $f(t) = (2^{100})^{(t/100)}$ indicate?

The quantity increases by a factor of 2^{100} every century.

23. On June 1, a fast-growing species of algae is accidentally introduced into a lake in a city park. It starts to grow and cover the surface of the lake in such a way that the area it covers doubles every day. If it continues to grow unabated, the lake will be totally covered and the fish in the lake will suffocate. At the rate it is growing, this will happen on June 30.

   a. Write an explicit formula for the sequence that models the percentage, p, of the surface area of the lake that is covered in algae t days after the algae was introduced. Let the initial percentage be P_0.

   $p(t) = P_0(2)^t$

   b. When will the lake be covered halfway?

   June 29 or June 28.

   c. On June 26, a pedestrian who walks by the lake every day warns that the lake will be completely covered soon. Her friend just laughs. Why might her friend be skeptical of the warning?

   Working backwards from June 30, on June 26 the lake was only 6.25% covered.

   d. On June 29, a cleanup crew arrives at the lake and removes almost all of the algae. When they are done, only 1% of the surface is covered with algae. How well does this solve the problem of the algae in the lake?

   Not well at all. Starting at 1% and doubling, we will arrive at 100% in less than one week.

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24. Zebra mussels are an invasive, fingernail-sized mollusk that are infesting some of the freshwater lakes in North America. Their population increases on average at a rate of 5% per day. Let 700 be the amount of zebra mussels in a certain lake at time $d = 0$ days. Which of the following functions g models the amount of zebra mussels after t weeks where 700 is the amount of zebra mussels at time $t = 0$?

B

(A) $g(t) = 700(1.05)^{(t/7)}$

(B) $g(t) = 700(1.05)^{7t}$

(C) $g(t) = 700\left(1.05^{(1/7)}\right)^{(7t)}$

(D) $g(t) = 700\left(1.05^{(7)}\right)^{(t/7)}$

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25. Inflation is affecting the cost of goods and services in an economy at a rate of 0.667% per month. Let the cost of a particular good be worth \$100 at time $t = 0$ years. Which of the following functions gives the value of this good t years later?

C

(A) $a(t) = 100(1.667^{12})^t$

(B) $b(t) = 100\left(1 + 0.667^{(12)}\right)^t$

(C) $c(t) = 100\left(1.0067^{(12)}\right)^t$

(D) $d(t) = 100\left(1 + 0.00667^{(12)}\right)^t$