AP Precalc

Write your questions and thoughts here!

2.8 Inverse Functions

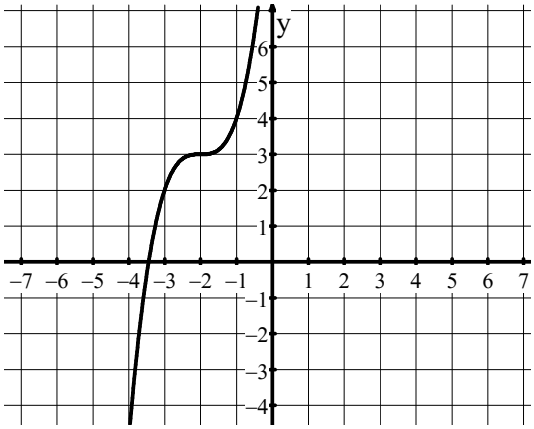
2.8 Notes

A function, f, has an inverse function, or is _____, if each output value of f is mapped from a unique input value.

$$f(x) = (x+2)^3 + 3$$

x	$f(x)$

x	$f^{-1}(x)$

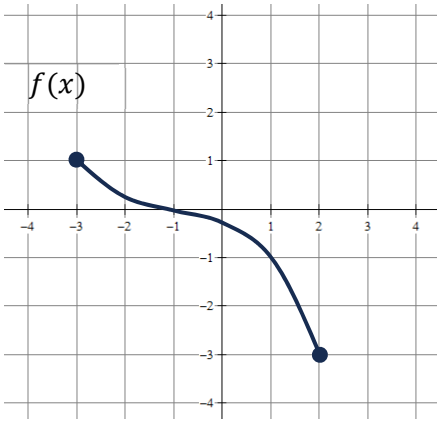


An inverse function is a reverse mapping of the function. That is, if $f(a) = b$, then $f^{-1}(b) = a$. Another way of thinking of this is if a function has the coordinate pair (a, b), then the inverse function has the coordinate pair (b, a)

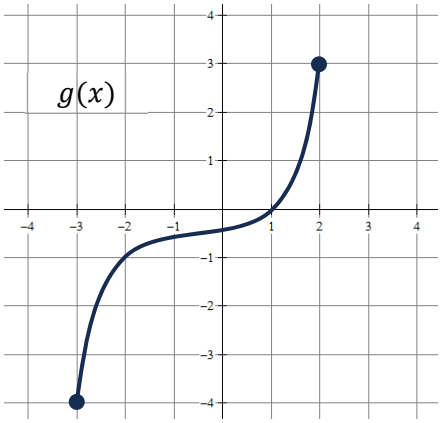
If the function is increasing or decreasing only, then it is invertible. If the graph "turns" (has a max or min), then it is no longer invertible because there will be output values that are the same for different input values. Think of this as a "horizontal line test". The **Vertical Line Test** checks to see if a graph is a function. The **Horizontal Line Test** checks to see if a graph's inverse is a function.

Are the following functions invertible? Sketch the graph of the inverse.

1.



2.

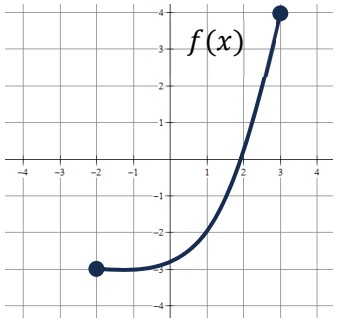


The inverse of the graph of the function $f(x)$ can be found by reversing the roles of the x- and y-axes. This means we can reflect the graph of f over the line $y = x$ to get the graph of the inverse.

The domain and range of a function and its inverse are swapped.

What is the minimum value of $f^{-1}(x)$?

What is the maximum value of $f^{-1}(x)$?



Write your questions and thoughts here!

One method of finding the inverse function is to reverse the roles of x and y in the equation, then solve for y.

3. Find the inverse function of $f(x) = (x+2)^3 + 3$.

The domain of a function can be restricted to make the function invertible.

4. Find the inverse function of $f(x) = \frac{1}{2}x^2 + 2$

What is the domain and range of the inverse function?

Find the inverse function along with the domain and range of the inverse.

5. $f(x) = \sqrt{x-3} + 2$	6. $f(x) = \frac{3}{x+6}$	7. $f(x) = \frac{2x-3}{x+4}$
Domain of f^{-1}.	Domain of f^{-1}.	Domain of f^{-1}.
Range of f^{-1}.	Range of f^{-1}.	Range of f^{-1}.

Composition of f and f^{-1}

The composition of a function, f, and its inverse function f^{-1}, is the identity function.

$$f\left(f^{-1}(x)\right) =$$

8. Are $f(x) = \frac{2}{x+3}$ and $g(x) = \frac{2}{x} - 3$ inverses?

2.8 Inverse Functions

AP Precalculus

2.8 Practice

Find the inverse of each function and list the domain and range of $f^{-1}(x)$.

1. $f(x) = (x-3)^3 + 4$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

2. $f(x) = \frac{1}{7}x + 6$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

3. $f(x) = (x+1)^2 - 2$ for $x \geq -1$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

4. $f(x) = \sqrt{x+2} - 3$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

5. $f(x) = (x-2)^2 + 5$ for $x \leq 2$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

6. $f(x) = \frac{2}{x-1}$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

7. $f(x) = -(x+4)^2 - 1$ for $x \leq -4$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

8. $f(x) = -\sqrt{x+1} + 3$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

9. $f(x) = \frac{2x+5}{3x-4}$

Domain of $f^{-1}(x)$:

Range of $f^{-1}(x)$:

Use the tables below to find the given values.

10.

x	$f(x)$
1	-2
2	3
3	6
4	2
5	4
6	1

a. $f(1)$
b. $f(6)$
c. $f^{-1}(1)$
d. $f^{-1}(4)$
e. $f(2)$
f. $f^{-1}(6)$

11.

x	$f(x)$
-3	2
-1	7
2	10
7	-3
8	-1
10	8

a. $f(2)$
b. $f(10)$
c. $f^{-1}(7)$
d. $f^{-1}(-3)$
e. $f(7)$
f. $f^{-1}(2)$

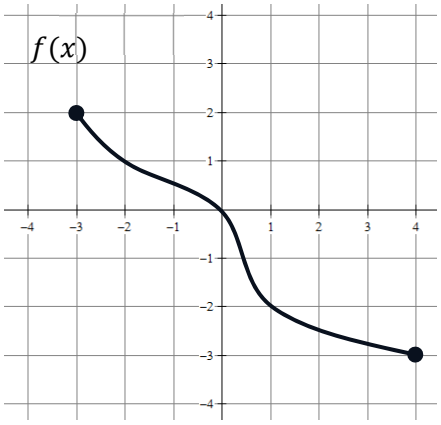
12.

x	$f(x)$
-10	-6
-6	7
-2	-10
3	11
7	3
11	-2

a. $f(-10)$
b. $f(3)$
c. $f^{-1}(7)$
d. $f^{-1}(-6)$
e. $f(7)$
f. $f^{-1}(-2)$

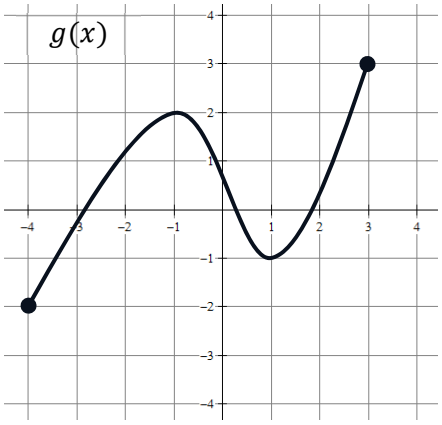
The graph of a function is given below. Identify if the function is invertible. Sketch the graph of the inverse regardless of whether or not it is invertible.

13.



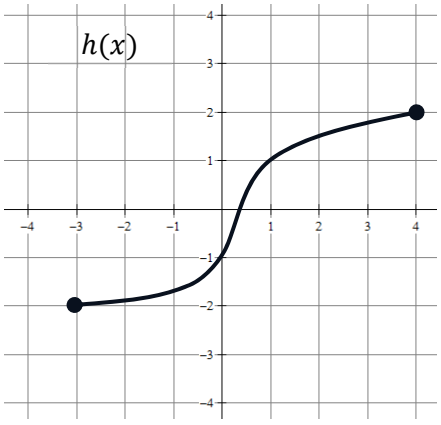
Is $f(x)$ invertible?

14.



Is $g(x)$ invertible?

15.



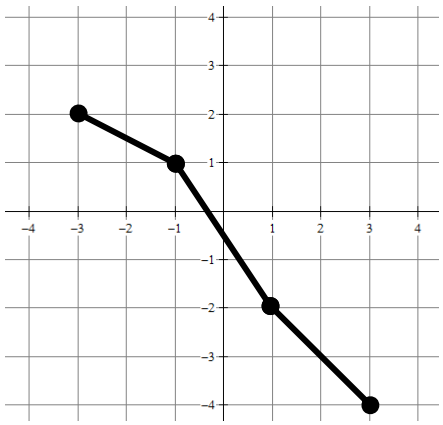
Is $h(x)$ invertible?

Determine if the two functions are inverses of each other using composition.

16. $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}x - \frac{5}{3}$

17. $f(x) = \sqrt[3]{3 - x}$ and $g(x) = x^3 - 3$

18. The graph of the piecewise-linear function f is shown in the figure. Let g be the inverse function of f. What is the minimum value of g?



Graph of f

(A) -4

(B) -3

(C) 2

(D) 3

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19. Mr. Brust is filling up his backdoor kiddie pool with the water hose. The amount of water, in gallons, in the pool t minutes after he turns on the water can be modeled by P, an increasing function of time t. Which of the following gives a verbal representation of the function P^{-1}, the inverse of P?

(A) P^{-1} is an increasing function of the amount of time after the water is turned on.

(B) P^{-1} is a decreasing function of the amount of time after the water is turned on.

(C) P^{-1} is an increasing function of the amount of water in the pool.

(D) P^{-1} is a decreasing function of the amount of water in the pool.

20.

x	1	2	3	4	5
$f(x)$	-18	-10	-3	1	26

Calculator active. Let f be an increasing function for $x \geq 0$. The table gives values of $f(x)$ at selected values of x. The function g is given by $g(x) = \frac{x^4+16x^3+50}{x-2}$

a. The function h is defined by $h(x) = (g \circ f)(x) = g\big(f(x)\big)$. Find the value of $h(2)$ as a decimal approximation or indicate that it is not defined.

b. Find the value of $f^{-1}(1)$, or indicate that it is not defined.