2.8 Inverse Functions

AP Precalculus

Solutions

2.8 Practice

Find the inverse of each function and list the domain and range of $f^{-1}(x)$.

1. $f(x) = (x-3)^3 + 4$

$x - 4 = (y-3)^3$

$\sqrt[3]{x-4} = y-3$

$f^{-1}(x) = \sqrt[3]{x-4} + 3$

Domain of $f^{-1}(x)$: $\mathbb{R}$

Range of $f^{-1}(x)$: $\mathbb{R}$

2. $f(x) = \frac{1}{7}x + 6$

D: $\mathbb{R}$ R: $\mathbb{R}$

$x - 6 = \frac{1}{7}y$

$f^{-1}(x) = 7x - 42$

Domain of $f^{-1}(x)$: $\mathbb{R}$

Range of $f^{-1}(x)$: $\mathbb{R}$

3. $f(x) = (x+1)^2 - 2$ for $x \geq -1$

$x + 2 = (y+1)^2$

$\pm\sqrt{x+2} = y+1$

D: $x \geq -1$ R: $y \geq -2$

$f^{-1}(x) = \sqrt{x+2} - 1$

Domain of $f^{-1}(x)$: $x \geq -2$

Range of $f^{-1}(x)$: $y \geq -1$

4. $f(x) = \sqrt{x+2} - 3$

D: $x \geq -2$ R: $y \geq -3$

$x + 3 = \sqrt{y+2}$

$(x+3)^2 = y+2$

$f^{-1}(x) = (x+3)^2 - 2$

Domain of $f^{-1}(x)$: $x \geq -3$

Range of $f^{-1}(x)$: $y \geq -2$

5. $f(x) = (x-2)^2 + 5$ for $x \leq 2$

$x - 5 = (y-2)^2$

$\pm\sqrt{x-5} = y-2$

D: $x \leq 2$ R: $y \geq 5$

$f^{-1}(x) = -\sqrt{x-5} + 2$

Domain of $f^{-1}(x)$: $x \geq 5$

Range of $f^{-1}(x)$: $y \leq 2$

6. $f(x) = \frac{2}{x-1}$

D: $\mathbb{R}, x \neq 1$ R: $\mathbb{R}, y \neq 0$

$x = \frac{2}{y-1}$

$x(y-1) = 2$

$xy - x = 2$

$xy = 2 + x$

$f^{-1}(x) = \frac{2}{x} + 1$

Domain of $f^{-1}(x)$: $\mathbb{R}, x \neq 0$

Range of $f^{-1}(x)$: $\mathbb{R}, y \neq 1$

7. $f(x) = -(x+4)^2 - 1$ for $x \leq -4$

$x + 1 = -(y+4)^2$

$-x - 1 = (y+4)^2$

$\pm\sqrt{-x-1} = y+4$

D: $x \leq -4$ R: $y \leq -1$

$f^{-1}(x) = -\sqrt{-x-1} - 4$

Domain of $f^{-1}(x)$: $x \leq -1$

Range of $f^{-1}(x)$: $y \leq -4$

8. $f(x) = -\sqrt{x+1} + 3$

$x - 3 = -\sqrt{y+1}$

$(x-3)^2 = y+1$

D: $x \geq -1$ R: $y \leq 3$

$f^{-1}(x) = (x-3)^2 - 1$

Domain of $f^{-1}(x)$: $x \leq 3$

Range of $f^{-1}(x)$: $y \geq -1$

9. $f(x) = \frac{2x+5}{3x-4}$

D: $\mathbb{R}, x \neq \frac{4}{3}$ R: $\mathbb{R}, y \neq \frac{2}{3}$

$x(3y-4) = 2y+5$

$3xy - 4x = 2y + 5$

$3xy - 2y = 4x + 5$

$y(3x-2) = 4x+5$

$f^{-1}(x) = \frac{4x+5}{3x-2}$

Domain of $f^{-1}(x)$: $\mathbb{R}, x \neq \frac{2}{3}$

Range of $f^{-1}(x)$: $\mathbb{R}, y \neq \frac{4}{3}$

Use the tables below to find the given values.

10.

x	$f(x)$
1	-2
2	3
3	6
4	2
5	4
6	1

a. $f(1)$ -2

b. $f(6)$ 1

c. $f^{-1}(1)$ 6

d. $f^{-1}(4)$ 5

e. $f(2)$ 3

f. $f^{-1}(6)$ 3

11.

x	$f(x)$
-3	2
-1	7
2	10
7	-3
8	-1
10	8

a. $f(2)$ 10

b. $f(10)$ 8

c. $f^{-1}(7)$ -1

d. $f^{-1}(-3)$ 7

e. $f(7)$ -3

f. $f^{-1}(2)$ -3

12.

x	$f(x)$
-10	-6
-6	7
-2	-10
3	11
7	3
11	-2

a. $f(-10)$ -6

b. $f(3)$ 11

c. $f^{-1}(7)$ -6

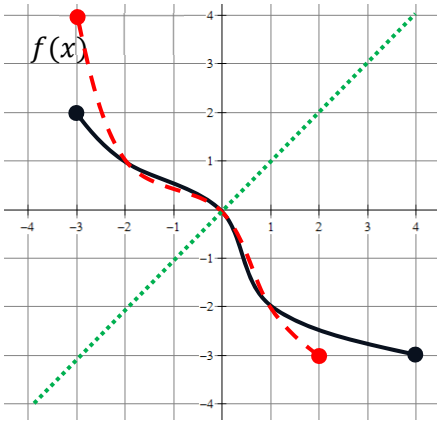
d. $f^{-1}(-6)$ -10

e. $f(7)$ 3

f. $f^{-1}(-2)$ 11

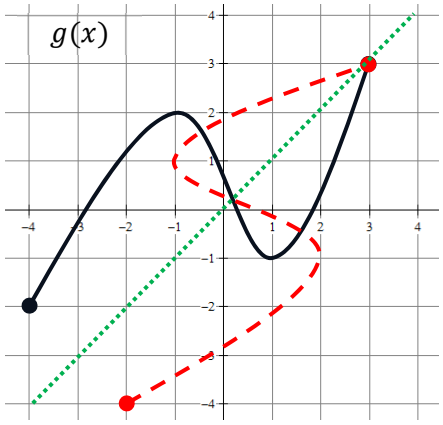
The graph of a function is given below. Identify if the function is invertible. Sketch the graph of the inverse regardless of whether or not it is invertible.

13.



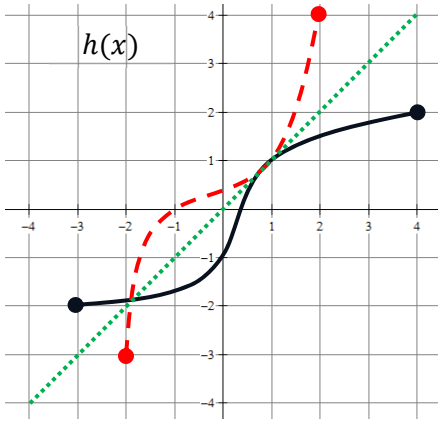
Is $f(x)$ invertible? yes

14.



Is $g(x)$ invertible? no

15.



Is $h(x)$ invertible? yes

Determine if the two functions are inverses of each other using composition.

16. $f(x) = 3x + 5$ and $g(x) = \frac{1}{3}x - \frac{5}{3}$

$f(g(x))$

$3\left(\frac{1}{3}x - \frac{5}{3}\right) + 5$

$x - 5 + 5$

x ✓

$g(f(x))$

$\frac{1}{3}(3x+5) - \frac{5}{3}$

$x + \frac{5}{3} - \frac{5}{3}$

x ✓

Yes!

17. $f(x) = \sqrt[3]{3 - x}$ and $g(x) = x^3 - 3$

$f(g(x))$

$\sqrt[3]{3 - (x^3 - 3)}$

$\sqrt[3]{6 - x^3}$

$g(f(x))$

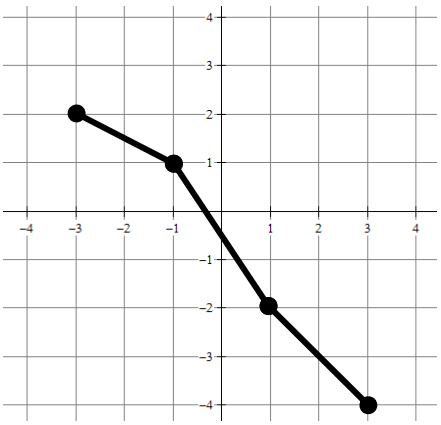
$\left(\sqrt[3]{3-x}\right)^3 - 3$

$3 - x - 3$

$-x$

No!

18. The graph of the piecewise-linear function f is shown in the figure. Let g be the inverse function of f. What is the minimum value of g?



Graph of f

(A) -4

(B) -3

(C) 2

(D) 3

B

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19. Mr. Brust is filling up his backdoor kiddie pool with the water hose. The amount of water, in gallons, in the pool t minutes after he turns on the water can be modeled by P, an increasing function of time t. Which of the following gives a verbal representation of the function P^{-1}, the inverse of P?

(A) P^{-1} is an increasing function of the amount of time after the water is turned on.

(B) P^{-1} is a decreasing function of the amount of time after the water is turned on.

(C) P^{-1} is an increasing function of the amount of water in the pool.

(D) P^{-1} is a decreasing function of the amount of water in the pool.

20.

x	1	2	3	4	5
$f(x)$	-18	-10	-3	1	26

Calculator active. Let f be an increasing function for $x \geq 0$. The table gives values of $f(x)$ at selected values of x. The function g is given by $g(x) = \frac{x^4+16x^3+50}{x-2}$

a. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(2)$ as a decimal approximation or indicate that it is not defined.

$g(f(2)) = g(-10) \approx 495.833$

b. Find the value of $f^{-1}(1)$, or indicate that it is not defined.

$f^{-1}(1) = 4$