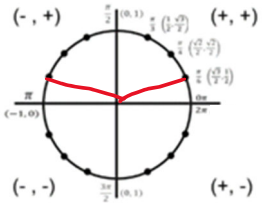
3.10 Trigonometric Equations and Inequalities

AP Precalculus

3.10 Practice

Solve each equation for $0 \le x \le 2\pi$. Find the exact value(s) using the unit circle.

1. $2\sin x + 3 = 4$



$-3 \quad -3$

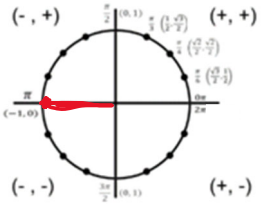
$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

2. $4 - 3\cos x = 7$



$-4 \quad -4$

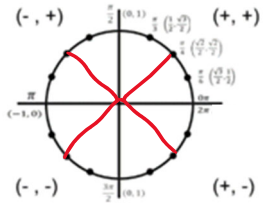
$\frac{-3\cos x}{-3} = \frac{3}{-3}$

$\cos x = -1$

$x = \cos^{-1}(-1)$

$x = \pi$

3. $\tan^2 x = 1$



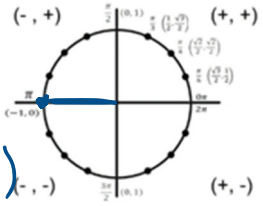
$\sqrt{\tan^2 x} = \pm\sqrt{1}$

$\tan x = \pm 1$

$x = \tan^{-1}(\pm 1)$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4. $-1 = \cos^2 x + 2\cos x$



$+1 \quad +1$

$0 = \cos^2 x + 2\cos x + 1$

$0 = (\cos x + 1)(\cos x + 1)$

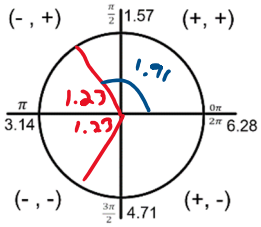
$\cos x + 1 = 0 \quad | \quad \cos x + 1 = 0$

$x = \cos^{-1}(-1) \quad | \quad x = \cos^{-1}(-1)$

$x = \pi$

Solve each equation for $0 \le \theta \le 2\pi$. Find the approximate value(s) using a calculator.

5. $6 = 3\cos\theta + 7$



$-7 \quad -7$

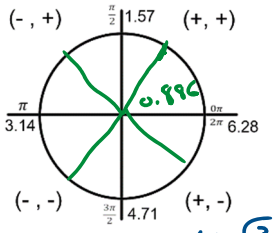
$\frac{-1}{3} = \frac{3\cos\theta}{3}$

$-\frac{1}{3} = \cos\theta$

$\theta = \cos^{-1}\left(-\frac{1}{3}\right)$

$\theta \approx 1.910$ and 4.372

6. $5\sin^2\theta + 3 = 6$



$-3 \quad -3$

$\frac{5\sin^2\theta}{5} = \frac{3}{5}$

$\sin^2\theta = \frac{3}{5}$

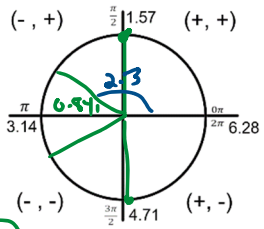
$\sqrt{\sin^2\theta} = \pm\sqrt{\frac{3}{5}}$

$\sin\theta = \pm\sqrt{\frac{3}{5}}$

$\theta = \sin^{-1}\left(\pm\sqrt{\frac{3}{5}}\right)$

$\theta = 0.896, 4.037$
$2.255, 5.397$

7. $6\cos^2\theta + 4\cos\theta = 0$



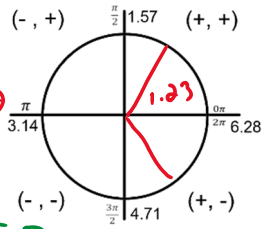
$2\cos\theta(3\cos\theta + 2) = 0$

$2\cos\theta = 0 \quad | \quad 3\cos\theta + 2 = 0$

$\theta = \cos^{-1}(0) \quad | \quad \theta = \cos^{-1}\left(-\frac{2}{3}\right)$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, 2.300, 3.982$

8. $\cos\theta = 3\cos^2\theta$



$-\cos\theta \quad -\cos\theta$

$0 = 3\cos^2\theta - \cos\theta$

$0 = \cos\theta(3\cos\theta - 1)$

$\cos\theta = 0 \quad | \quad 3\cos\theta - 1 = 0$

$\theta = \cos^{-1}(0) \quad | \quad \theta = \cos^{-1}\left(\frac{1}{3}\right)$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}, 1.230, 5.052$

Solve each equation. Find ALL exact value(s) using the unit circle.

9. $2\sin^2\theta = \sin\theta$

$-\sin\theta \quad -\sin\theta$

$2\sin^2\theta - \sin\theta = 0$

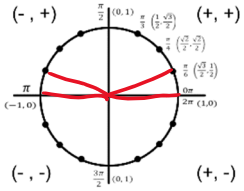
$\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0 \quad | \quad 2\sin\theta - 1 = 0$

$\theta = \sin^{-1}(0) \quad | \quad \theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 0\pi + \pi n$ and $\frac{\pi}{6} + 2\pi n$, $\frac{5\pi}{6} + 2\pi n$

where n is an integer



10. $4\tan^2(2x) = 12$

$\frac{4}{4} \quad \frac{12}{4}$

$\sqrt{\tan^2(2x)} = \pm\sqrt{3}$

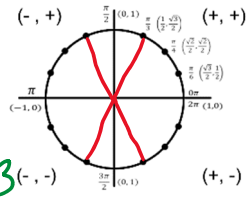
$\tan(2x) = \pm\sqrt{3}$

$2x = \tan^{-1}(\pm\sqrt{3})$

$\frac{2x}{2} = \frac{\pi}{3} + \pi n$ and $\frac{2\pi}{3} + \pi n$ $\quad 2$

$x = \frac{\pi}{6} + \frac{\pi}{2}n$ and $\frac{\pi}{3} + \frac{\pi}{2}n$

where n is an integer



Solve each equation. Find ALL approximate value(s) using a calculator.

11. $7\sin^2 x = 5$

$\frac{7}{7}$

$\sqrt{\sin^2 x} = \sqrt{\frac{5}{7}}$

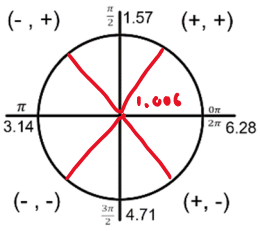
$\sin x = \pm\sqrt{\frac{5}{7}}$

$x = \sin^{-1}\left(\pm\sqrt{\frac{5}{7}}\right)$

$x = 1.006 + \pi n$

$-1.006 + \pi n$

where n is an integer



12. $5\sin\theta + 3 = 4$

$-3 \quad -3$

$\frac{5\sin\theta}{5} = \frac{1}{5}$

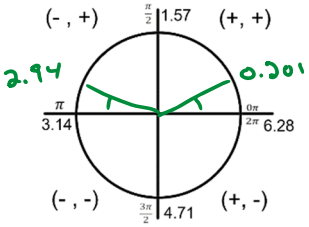
$\sin\theta = \frac{1}{5}$

$\theta = \sin^{-1}\left(\frac{1}{5}\right)$

$\theta = 0.201 + 2\pi n$

$2.940 + 2\pi n$

where n is an integer



Solve each inequality for $0 \le x \le 2\pi$. Find the exact value(s). Include a rough sketch.

13. $2\sin x + 2 > 3$

$-2 \quad -2$

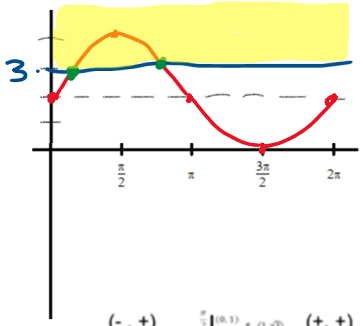
$\frac{2\sin x}{2} > \frac{1}{2}$

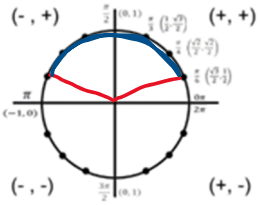
$\sin x > \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ and $\frac{5\pi}{6}$

$\frac{\pi}{6} < x < \frac{5\pi}{6}$





14. $2\cos x \ge \sqrt{3}$

$\frac{2}{2} \quad \frac{\sqrt{3}}{2}$

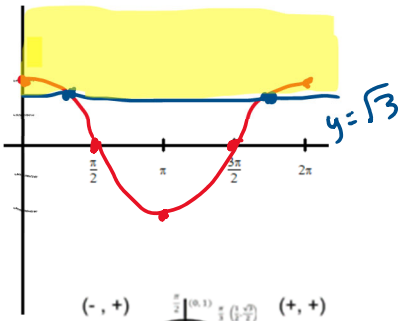
$\cos x \ge \frac{\sqrt{3}}{2}$

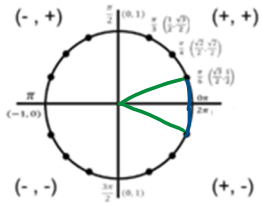
$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$x = \frac{\pi}{6}$ and $\frac{11\pi}{6}$

$0 \le x \le \frac{\pi}{6}$

$\frac{11\pi}{6} \le x \le 2\pi$





15. $2\cos\left(x - \frac{\pi}{2}\right) - 2 \le -3$

$+2 \quad +2$

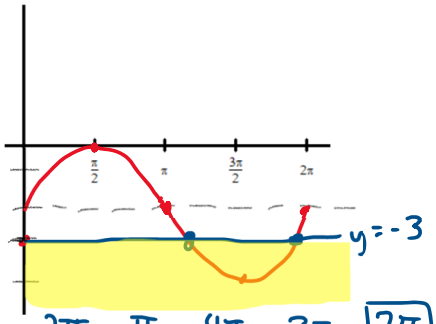
$\frac{2\cos\left(x - \frac{\pi}{2}\right)}{2} \le \frac{-1}{2}$

$\cos\left(x - \frac{\pi}{2}\right) \le -\frac{1}{2}$

$x - \frac{\pi}{2} = \cos^{-1}\left(-\frac{1}{2}\right)$

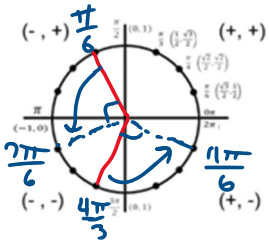
$x - \frac{\pi}{2} = \frac{2\pi}{3}$ and $\frac{4\pi}{3}$

$+\frac{\pi}{2} \quad +\frac{\pi}{2} \quad +\frac{\pi}{2}$



$\frac{2\pi}{3} + \frac{\pi}{2} = \frac{4\pi}{6} + \frac{3\pi}{6} = \frac{7\pi}{6}$

$\frac{4\pi}{3} + \frac{\pi}{2} = \frac{8\pi}{6} + \frac{3\pi}{6} = \frac{11\pi}{6}$



$\frac{7\pi}{6} \le x \le \frac{11\pi}{6}$

3.10 Trigonometric Equations and Inequalities

3.10 Test Prep

16. What are all values of θ, for $0 \le \theta \le 2\pi$, where $4\sin^2\theta = 2\sin\theta$?

(A) $0, \frac{\pi}{3}, \pi, \frac{2\pi}{3}, 2\pi$

(B) $0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}, 2\pi$

(C) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

(D) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

$-2\sin\theta \quad -2\sin\theta$

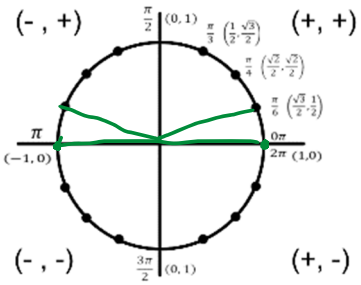
$4\sin^2\theta - 2\sin\theta = 0$

$2\sin\theta(2\sin\theta - 1) = 0$

$2\sin\theta = 0 \quad | \quad 2\sin\theta - 1 = 0$

$\theta = \sin^{-1}(0) \quad | \quad \theta = \sin^{-1}(\frac{1}{2})$

0 and π | $\frac{\pi}{6}$ and $\frac{5\pi}{6}$



17. The function f is given by $f(x) = 2\cos(3x) - 1$. For how many values of x where $0 \le x \le 2\pi$ does $f(x) = 0$?

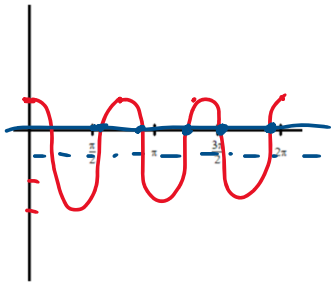
(A) None

(B) Two

(C) Four

(D) Six

period $= \frac{2\pi}{3}$



18. The function g is given by $g(x) = 2\cos(x)$. What are all solutions $g(x) = \sqrt{3}$?

(A) $x = \frac{\pi}{6} + 2\pi k$ and $\frac{5\pi}{6} + 2\pi k$, where k is any integer

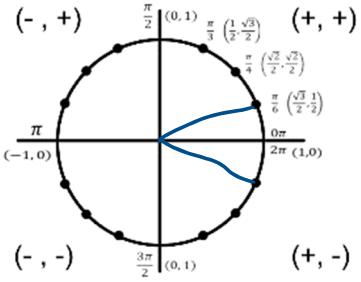
(B) $x = \pm\frac{\pi}{6} + 2\pi k$, where k is any integer

(C) $x = \frac{\pi}{3} + 2\pi k$ and $\frac{2\pi}{3} + 2\pi k$, where k is any integer

(D) $x = \pm\frac{\pi}{3} + 2\pi k$, where k is any integer

$\frac{\sqrt{3}}{2} = \frac{2\cos x}{2}$

$x = \cos^{-1}(\frac{\sqrt{3}}{2})$



19. What is the solution set for $4 < 2\sin(x) + 3$ where $0 \le x \le 2\pi$?

(A) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

(B) $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{5\pi}{6}, 2\pi\right)$

(C) $\left(\frac{\pi}{3}, \frac{2\pi}{3}\right)$

(D) $\left(0, \frac{\pi}{3}\right) \cup \left(\frac{2\pi}{3}, 2\pi\right)$

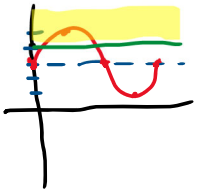
$-3 \quad -3$

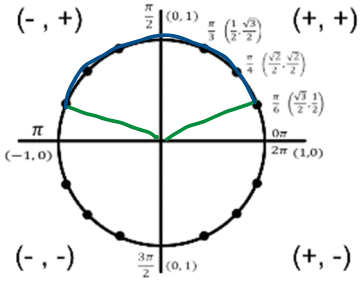
$\frac{1}{2} < \frac{2\sin x}{2}$

$\frac{1}{2} < \sin x$

$\sin^{-1}(\frac{1}{2}) = x$

$\frac{\pi}{6}$ and $\frac{5\pi}{6}$





20. What are all values of θ, for $0 \le \theta \le 2\pi$, where $\sin^2\theta - \cos\theta\sin\theta = 0$?

(A) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(B) $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

(C) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(D) $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

$\sin\theta(\sin\theta - \cos\theta) = 0$

$\sin\theta = 0 \quad | \quad \sin\theta - \cos\theta = 0$

$\theta = \sin^{-1}(0) \quad | \quad \sin\theta = \cos\theta$

0 and 2π and π | $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

$\sin = \cos$