3.12B Equivalent Representations of Trig Functions

AP Precalculus

3.12B Practice

Find the exact value of the sum or difference.

1. $\sin\left(\frac{5\pi}{4}+\frac{2\pi}{3}\right)$

$\sin\frac{5\pi}{4}\cos\frac{2\pi}{3}+\cos\frac{5\pi}{4}\sin\frac{2\pi}{3}$

$\left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{\sqrt{2}}{4}+\frac{-\sqrt{6}}{4}$

$\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

2. $\cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)$

$\cos\frac{3\pi}{4}\cos\frac{\pi}{3}-\sin\frac{3\pi}{4}\sin\frac{\pi}{3}$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)-\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$

$\boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

3. $\cos\left(\frac{5\pi}{4}-\frac{\pi}{6}\right)$

$\cos\frac{5\pi}{4}\cos\frac{\pi}{6}+\sin\frac{5\pi}{4}\sin\frac{\pi}{6}$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\frac{-\sqrt{6}}{4}+\frac{-\sqrt{2}}{4}$

$\boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$

4. $\sin\left(\frac{3\pi}{2}-\frac{4\pi}{3}\right)$

$\sin\frac{3\pi}{2}\cos\frac{4\pi}{3}-\sin\frac{4\pi}{3}\cos\frac{3\pi}{2}$

$(-1)\left(-\frac{1}{2}\right)-\left(-\frac{\sqrt{3}}{2}\right)(0)$

$\frac{1}{2}-0$

$\boxed{\frac{1}{2}}$

5. $\cos\left(\frac{\pi}{2}-\frac{4\pi}{3}\right)$

$\cos\frac{\pi}{2}\cos\frac{4\pi}{3}+\sin\frac{\pi}{2}\sin\frac{4\pi}{3}$

$(0)\left(-\frac{1}{2}\right)+(1)\left(-\frac{\sqrt{3}}{2}\right)$

$0+\frac{-\sqrt{3}}{2}$

$\boxed{-\frac{\sqrt{3}}{2}}$

6. $\sin\left(\frac{11\pi}{6}+\pi\right)$

$\sin\frac{11\pi}{6}\cos\pi+\sin\pi\cos\frac{11\pi}{6}$

$\left(-\frac{1}{2}\right)(-1)+(0)\left(\frac{\sqrt{3}}{2}\right)$

$\frac{1}{2}+0$

$\boxed{\frac{1}{2}}$

Simplify the following.

7. $\sin\left(x-\frac{5\pi}{3}\right)$

$\sin x\cos\frac{5\pi}{3}-\sin\frac{5\pi}{3}\cos x$

$\sin x\left(\frac{1}{2}\right)-\left(-\frac{\sqrt{3}}{2}\right)\cos x$

$\frac{1}{2}\sin x+\frac{\sqrt{3}}{2}\cos x$

8. $\cos\left(\frac{3\pi}{4}+\theta\right)$

$\cos\frac{3\pi}{4}\cos\theta-\sin\frac{3\pi}{4}\sin\theta$

$\left(-\frac{\sqrt{2}}{2}\right)\cos\theta-\left(\frac{\sqrt{2}}{2}\right)\sin\theta$

$-\frac{\sqrt{2}}{2}\cos\theta-\frac{\sqrt{2}}{2}\sin\theta$

9. $4\sin(2x)$

$4[2\sin x\cos x]$

$8\sin x\cos x$

Use trig identities to solve the trig equations for $0\le x\le 2\pi$. Find exact values.

10. $\sin(2x)+\sin x=0$

$2\sin x\cos x+\sin x=0$

$\sin x(2\cos x+1)=0$

$\sin x=0$ | $\cos x=-\frac{1}{2}$

$x=0,\pi,2\pi$ $\quad x=\frac{2\pi}{3},\frac{4\pi}{3}$

11. $\sin(2x)\sec x=0$

$2\sin x\cdot\cos x\cdot\frac{1}{\cos x}=0$

$\frac{2\sin x}{2}=\frac{0}{2}$

$\sin x=0$

$x=0,\pi,2\pi$

12. $\cos(2x)=\cos^2 x$

$2\cos^2x-1=\cos^2x$

$-\cos^2x \quad -\cos^2x$

$\cos^2x-1=0$

$\pm1 \quad \pm1$

$\sqrt{\cos^2x}=\sqrt{1}$

$\cos x=\pm1$

$x=0,\pi,2\pi$

13. $\cos(2x)+\sin^2 x=0$

$1-2\sin^2x+\sin^2x=0$

$1-\sin^2x=0$

$\sqrt{\cos^2x}=\sqrt{0}$

$\cos x=0$

$x=\frac{\pi}{2},\frac{3\pi}{2}$

14. The function f is given by $f(\theta) = \cos(2\theta)$. Which of the following expressions is equivalent to $f(\theta)$?

(A) $2\cos\theta$

(B) $1 - 2\sin^2\theta$

(C) $1 - 2\cos^2\theta$

(D) $2\sin\theta\cos\theta$

15. The function g is defined by $g(x) = \sin\left(x + \frac{\pi}{3}\right)$. The solutions to which of the following equations on the interval $0 \le x \le 2\pi$ are the solutions to $g(x) = 1$ on the interval $0 \le x \le 2\pi$?

(A) $\sin x + \sqrt{3}\cos x = 2$

(B) $\sin x - \sqrt{3}\cos x = 2$

(C) $\sqrt{3}\sin x + \cos x = 2$

(D) $\sqrt{3}\sin x - \cos x = 2$

$1 = \sin x \cos\frac{\pi}{3} + \cos x \sin\frac{\pi}{3}$

$1 = \sin x\left(\frac{1}{2}\right) + \cos x\left(\frac{\sqrt{3}}{2}\right)$

$2\left[1 = \frac{1}{2}\sin x + \frac{\sqrt{3}}{2}\cos x\right]$

$2 = \sin x + \sqrt{3}\cos x$

16. The function f is given by $f(\theta) = \cos\left(\theta + \frac{3\pi}{2}\right)$. What is the value of $f\left(\frac{\pi}{3}\right)$?

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $-\frac{\sqrt{3}}{2}$

$f\left(\frac{\pi}{3}\right) = \cos\left(\frac{\pi}{3} + \frac{3\pi}{2}\right)$

$= \cos\frac{\pi}{3}\cos\frac{3\pi}{2} - \sin\frac{\pi}{3}\sin\frac{3\pi}{2}$

$\left(\frac{1}{2}\right)(0) - \left(\frac{\sqrt{3}}{2}\right)(-1)$

$\frac{\sqrt{3}}{2}$

quad II

17. Given $\cos(2x) = \frac{3}{5}$ and $\frac{\pi}{2} \le x \le \pi$, what is the exact value of $\sin(x)$?

(A) $\frac{2\sqrt{5}}{5}$

(B) $\frac{\sqrt{5}}{5}$

(C) $-\frac{\sqrt{5}}{5}$

(D) $-\frac{2\sqrt{5}}{5}$

$1 - 2\sin^2 x = \frac{3}{5}$

$-1 \qquad -1$

$\frac{-2\sin^2 x}{-2} = \frac{-2}{5}$

$\sqrt{\sin^2 x} = \sqrt{\frac{1}{5}}$

$\sin x = \pm\frac{1}{\sqrt{5}}$

$\sin x = \pm\frac{1}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}} = \pm\frac{\sqrt{5}}{5}$

sin is positive in quad II