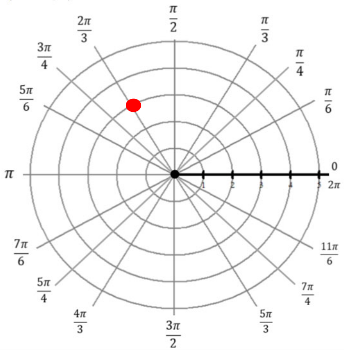
3.13 Trigonometry and Polar Coordinates

AP Precalculus

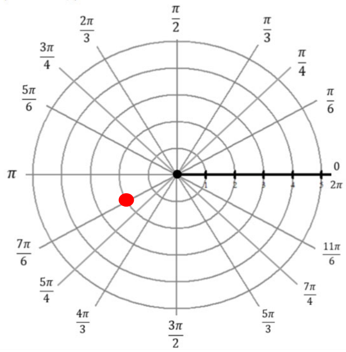
3.13 Practice

Plot the following polar points.

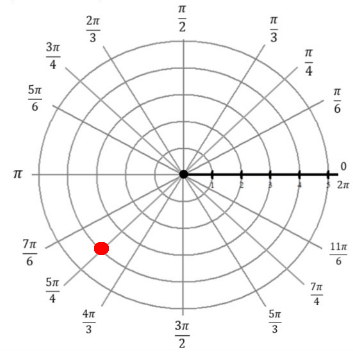
1. $\left(3, \frac{2\pi}{3}\right)$



2. $\left(-2, \frac{\pi}{6}\right)$

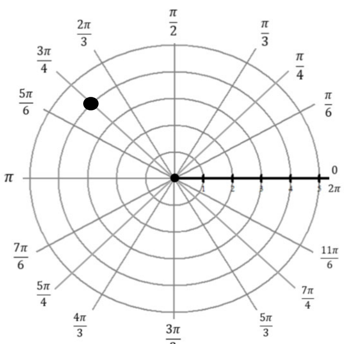


3. $\left(4, -\frac{3\pi}{4}\right)$



Name the polar point four different ways.

4.



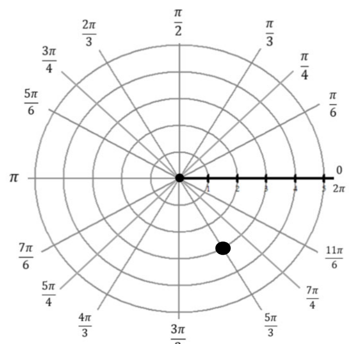
$\left(4, \frac{3\pi}{4}\right)$

$\left(4, -\frac{5\pi}{4}\right)$

$\left(-4, \frac{7\pi}{4}\right)$

$\left(-4, -\frac{\pi}{4}\right)$

5.



$\left(3, \frac{5\pi}{3}\right)$

$\left(3, -\frac{\pi}{3}\right)$

$\left(-3, \frac{2\pi}{3}\right)$

$\left(-3, -\frac{4\pi}{3}\right)$

Convert the polar coordinates to rectangular coordinates.

6. $\left(4, \frac{2\pi}{3}\right)$

$x = 4\cos\left(\frac{2\pi}{3}\right)$

$4\left(-\frac{1}{2}\right) = -2$

$y = 4\sin\left(\frac{2\pi}{3}\right)$

$4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$\boxed{(-2, 2\sqrt{3})}$

7. $\left(-2, \frac{\pi}{4}\right)$

$x = -2\cos\left(\frac{\pi}{4}\right)$

$-2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$y = -2\sin\left(\frac{\pi}{4}\right)$

$-2\left(\frac{\sqrt{2}}{2}\right) = -\sqrt{2}$

$\boxed{(-\sqrt{2}, -\sqrt{2})}$

8. $\left(5, \frac{5\pi}{6}\right)$

$x = 5\cos\left(\frac{5\pi}{6}\right)$

$5\left(-\frac{\sqrt{3}}{2}\right) = \frac{-5\sqrt{3}}{2}$

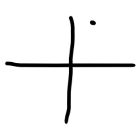
$y = 5\sin\left(\frac{5\pi}{6}\right)$

$5\left(\frac{1}{2}\right) = \frac{5}{2}$

$\boxed{\left(-\frac{5\sqrt{3}}{2}, \frac{5}{2}\right)}$

Convert the rectangular coordinates to polar coordinates where $0 \leq \theta \leq 2\pi$.

9. $(2, 4)$



$r = \sqrt{2^2 + 4^2}$

$\sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$\theta = \tan^{-1}\left(\frac{4}{2}\right)$

1.107

$\boxed{(2\sqrt{5}, 1.107)}$

10. $(-3, 4)$

$r = \sqrt{(-3)^2 + 4^2}$

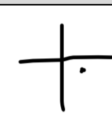
$\sqrt{9+16} = \sqrt{25} = 5$

$\theta = \tan^{-1}\left(\frac{4}{-3}\right) + \pi$

$-0.927 + \pi = 2.214$

$\boxed{(5, 2.214)}$

11. $(2, -1)$



$r = \sqrt{2^2 + (-1)^2}$

$\sqrt{4+1} = \sqrt{5}$

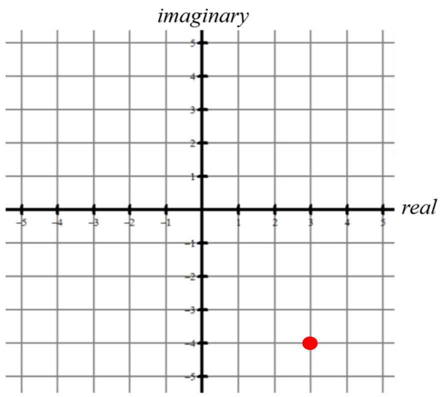
$\theta = \tan^{-1}\left(\frac{-1}{2}\right) + 2\pi$

$-0.463 + 2\pi = 5.819$

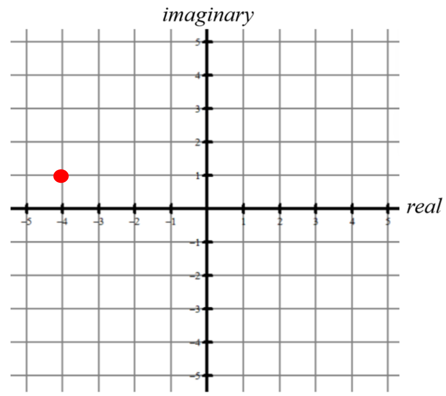
$\boxed{(\sqrt{5}, 5.819)}$

Plot the following points on the complex plane.

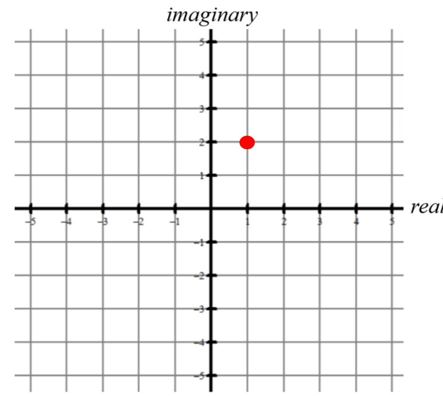
12. $3-4i$



13. $-4+i$



14. $1+2i$



Convert rectangular complex numbers to polar form.

15. $3+4i$

$r=\sqrt{3^2+4^2}$

$\sqrt{9+16}=\sqrt{25}=5$

$\theta=\tan^{-1}\left(\frac{4}{3}\right)$

0.927

$\boxed{5\left[\cos(0.927)+i\sin(0.927)\right]}$

16. $2-5i$

$r=\sqrt{2^2+(-5)^2}$

$\sqrt{4+25}=\sqrt{29}$

$\theta=\tan^{-1}\left(\frac{-5}{2}\right)$

-1.190

$\boxed{\sqrt{29}\left[\cos(-1.190)+i\sin(-1.190)\right]}$

17. $-3-3i$

$r=\sqrt{(-3)^2+(-3)^2}$

$\sqrt{9+9}=\sqrt{18}=3\sqrt{2}$

$\theta=\tan^{-1}\left(\frac{-3}{-3}\right)+\pi$

$0.785+\pi=3.926$

$\boxed{3\sqrt{2}\left[\cos(3.926)+i\sin(3.926)\right]}$

Convert polar complex numbers to rectangular form.

18. $10\left[\cos\left(\frac{3\pi}{4}\right)+i\sin\left(\frac{3\pi}{4}\right)\right]$

$x=10\cos\left(\frac{3\pi}{4}\right)$

$10\left(-\frac{\sqrt{2}}{2}\right)=-5\sqrt{2}$

$y=10\sin\left(\frac{3\pi}{4}\right)$

$10\left(\frac{\sqrt{2}}{2}\right)=5\sqrt{2}$

$\boxed{-5\sqrt{2}+5i\sqrt{2}}$

19. $4\left[\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right]$

$x=4\cos\left(\frac{3\pi}{2}\right)$

$4(0)=0$

$y=4\sin\left(\frac{3\pi}{2}\right)$

$4(-1)=-4$

$\boxed{-4i}$

20. $3\cos\left(\frac{5\pi}{3}\right)+3i\sin\left(\frac{5\pi}{3}\right)$

$x=3\cos\left(\frac{5\pi}{3}\right)$

$3\left(\frac{1}{2}\right)=\frac{3}{2}$

$y=3\sin\left(\frac{5\pi}{3}\right)$

$3\left(-\frac{\sqrt{3}}{2}\right)=-\frac{3\sqrt{3}}{2}$

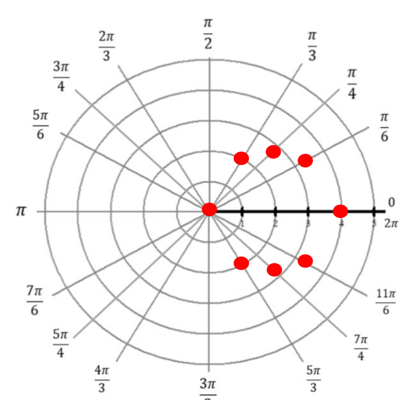
$\boxed{\frac{3}{2}-\frac{3i\sqrt{3}}{2}}$

Fill in the table with approximate decimal values using a calculator. Plot the points.

21. $r=4\cos\theta$

$\boldsymbol{\theta}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\boldsymbol{r}$	4	3.464	2.828	2	0

$\boldsymbol{\theta}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
$\boldsymbol{r}$	-2	-2.828	-3.464	-4



22. A complex number has the rectangular coordinates $(1, -\sqrt{3})$ Which of the following is one way to Express the complex number using its polar coordinates (r, θ) ?

   (A) $\left(2\cos\left(-\frac{\pi}{6}\right)\right) + i\left(2\sin\left(-\frac{\pi}{6}\right)\right)$

   (B) $\left(2\cos\left(-\frac{\pi}{3}\right)\right) + i\left(2\sin\left(-\frac{\pi}{3}\right)\right)$

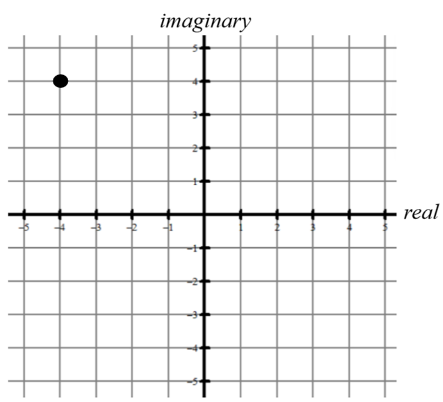
   (C) $\left(2\cos\left(\frac{\pi}{6}\right)\right) + i\left(2\sin\left(\frac{\pi}{6}\right)\right)$

   (D) $\left(2\cos\left(\frac{\pi}{3}\right)\right) + i\left(2\sin\left(\frac{\pi}{3}\right)\right)$

$1 - i\sqrt{3}$

$r = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = \frac{5\pi}{3}$ or $-\frac{\pi}{3}$



$-4 + 4i$

23. The graph above indicates a point in the complex plane. The complex number has polar coordinates (r, θ). Which of the following completes the expression for the point in polar form $(4\sqrt{2}\cos\theta) + i(4\sqrt{2}\sin\theta)$?

   (A) $\theta = \frac{\pi}{4}$

   (B) $\theta = \frac{3\pi}{4}$

   (C) $\theta = \frac{5\pi}{4}$

   (D) $\theta = \frac{7\pi}{4}$

$X = 4\sqrt{2}\cos\theta$

$\frac{-4}{4\sqrt{2}} = \frac{4\sqrt{2}\cos\theta}{4\sqrt{2}}$

$-\frac{1}{\sqrt{2}} = \cos\theta$

$-\frac{\sqrt{2}}{2} = \cos\theta$

$y = 4\sqrt{2}\sin\theta$

$\frac{4}{4\sqrt{2}} = \frac{4\sqrt{2}\sin\theta}{4\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \sin\theta$

$\frac{\sqrt{2}}{2} = \sin\theta$

$\frac{3\pi}{4}$ $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

24. The point A has polar coordinates $\left(4, \frac{7\pi}{6}\right)$. Which of the following also gives the location of point A in polar coordinates?

   (A) $\left(4, -\frac{11\pi}{6}\right)$

   (B) $\left(4, -\frac{5\pi}{6}\right)$

   (C) $\left(-4, -\frac{\pi}{6}\right)$

   (D) $\left(-4, -\frac{5\pi}{6}\right)$