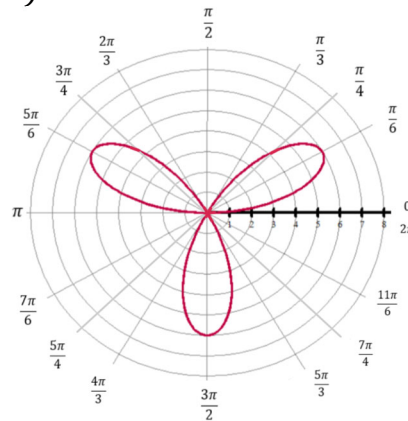
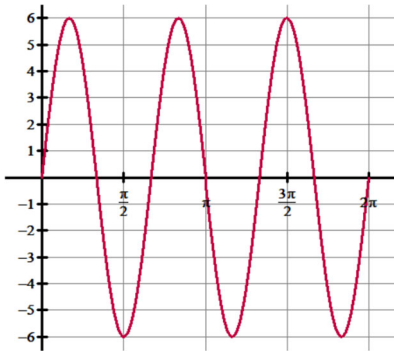


Write your questions
and thoughts here!Rate of Change of r **EXAMPLE #1:** $r = 6\sin(3\theta)$  r on the intervals:

$(0, \frac{\pi}{6})$

$(\frac{\pi}{6}, \frac{\pi}{3})$

$(\frac{\pi}{3}, \frac{\pi}{2})$

$(\frac{\pi}{2}, \frac{2\pi}{3})$

Distance from the Pole

	r is positive	r is negative
$r = f(\theta)$ is increasing		
$r = f(\theta)$ is decreasing		

EXAMPLE #2: $r = f(\theta)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
r	0	2.5	4.33	5	4.33	2.5	0

- Describe the behavior of the function on the interval $0 \leq \theta \leq \frac{\pi}{2}$.
- Describe the behavior of the function on the interval $\frac{\pi}{2} \leq \theta \leq \pi$.
- What is the average rate of change on the interval $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{3}$?
- Estimate the value of $f\left(\frac{\pi}{4}\right)$.

Write your questions
and thoughts here!

EXAMPLE #3: $r = f(\theta) = 3 + 5\sin(\theta)$ on the interval $\pi \leq \theta \leq 2\pi$

θ	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
r									

- Determine the interval(s) where f is increasing.
- Determine the interval(s) where f is decreasing.
- Is there at least one relative extrema on the interval $\pi \leq \theta \leq 2\pi$? Explain.
- The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\frac{5\pi}{4} \leq \theta \leq \frac{3\pi}{2}$. Justify your answer.
- The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\pi \leq \theta \leq \frac{7\pi}{6}$. Justify your answer.
- Find the average rate of change of f between $\theta = \frac{3\pi}{2}$ and $\theta = \frac{7\pi}{4}$.
- On what interval(s) must $f(\theta) = 0$.
- Compare the estimated value $f\left(\frac{5\pi}{3}\right)$ to the real value of $f\left(\frac{5\pi}{3}\right)$.

3.15 Rates of Change in Polar Functions

3.15 Practice

AP Precalculus

Use the table of selected values for the polar function $r = f(\theta)$ to answer the following.

1.

θ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
r	11	9.53	6	2.46	1	2.46	6	9.53	11

- a. Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.
- b. The distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\pi \leq \theta \leq 2\pi$. Justify your answer.
- c. Find the average rate of change of f between $\theta = \frac{5\pi}{4}$ and $\theta = \frac{7\pi}{4}$.
- d. Estimate the value of $f\left(\frac{\pi}{3}\right)$ using an average rate of change.
- e. Are there any extrema on the interval $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$? Explain how you know.

- 2.
- a. Is f increasing or decreasing on the interval $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$?
- b. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $0 \leq \theta \leq \frac{\pi}{4}$?
- c. Is the rate of change of f faster on the interval $\left[0, \frac{\pi}{8}\right]$ or the interval $\left[\frac{\pi}{8}, \frac{\pi}{4}\right]$? Justify.

θ	r
0	0
$\frac{\pi}{8}$	-1.41
$\frac{\pi}{4}$	-2
$\frac{3\pi}{8}$	-1.41
$\frac{\pi}{2}$	0

Use the polar function $r = f(\theta)$ to fill in the table and answer the questions. Calculator Active.

3. $r = f(\theta) = 8 \cos(\theta)$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{3\pi}{2}$	$\frac{11\pi}{6}$	2π
r									

- Determine the interval(s) where f is increasing. Determine the interval(s) where f is decreasing.
- How many extrema on the interval $\frac{5\pi}{6} \leq \theta \leq \frac{11\pi}{6}$?
- Determine the intervals where the distance between $f(\theta)$ and the pole is increasing on the interval $0 \leq \theta \leq 2\pi$. Justify your answer.
- Determine the intervals where the distance between $f(\theta)$ and the pole is decreasing on the interval $0 \leq \theta \leq 2\pi$. Justify your answer.
- Find the average rate of change of f between $\theta = \frac{\pi}{2}$ and $\theta = \frac{5\pi}{6}$. Use to estimate $f\left(\frac{2\pi}{3}\right)$.

4. $r = f(\theta) = -3 + 5 \sin(\theta)$

- Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$?
- Find the average rate of change of f between $\theta = \frac{\pi}{4}$ and $\theta = \frac{\pi}{2}$.
- Estimate the value of $f\left(\frac{5\pi}{6}\right)$ using an average rate of change.

θ	r
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	

3.15 Rates of Change in Polar Functions

5. Consider the graph of the polar function $r = f(\theta)$, where θ is increasing in the polar coordinate system on the interval $0 \leq \theta \leq 2\pi$. Given $f(\theta) < 0$ and decreasing on the interval $\pi \leq \theta \leq \frac{3\pi}{2}$ which of the following statements is true about the distance between the point with polar coordinates $(f(\theta), \theta)$ and the origin.
- (A) The distance is increasing for $0 \leq \theta \leq 2\pi$.
- (B) The distance is decreasing for $0 \leq \theta \leq 2\pi$.
- (C) The distance is increasing for $\pi \leq \theta \leq \frac{3\pi}{2}$.
- (D) The distance is decreasing for $\pi \leq \theta \leq \frac{3\pi}{2}$.

Use the table of selected values for the polar equation $r = f(\theta)$ below to answer questions 6 and 7.

CALCULATOR ACTIVE

θ	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$
r	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$

6. The graph of the polar function $r = f(\theta)$, is given the polar coordinate system. Which of the following descriptions is true ?
- (A) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.
- (B) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is increasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.
- (C) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is increasing.
- (D) As θ increasing from 0 to $\frac{\pi}{4}$, the polar function $r = f(\theta)$ is decreasing, and the distance between the point $(f(\theta), \theta)$ on the curve and the origin is decreasing.
7. If the value of $r = f\left(\frac{\pi}{8}\right)$ is estimated using the average rate of change of the function over the interval $0 \leq \theta \leq \frac{\pi}{4}$, which of the following is true?
- (A) The estimated value would be an overestimate of the actual value by approximately 0.116.
- (B) The estimated value would be an underestimate of the actual value by approximately -0.23.
- (C) The estimated value would be an overestimate of the actual value by approximately 1.616.
- (D) The estimated value would be an underestimate of the actual value by approximately -1.043.