3.6A Sinusoidal Function Transformations

AP Precalculus

Solutions

3.6A Practice

Identify the amplitude, period, and frequency of each function.

1. $f(\theta) = 13\sin(2\theta) - 7$

Amp: 13 Period: $\frac{2\pi}{2} = \pi$

Freq: $\frac{1}{\pi}$

2. $g(t) = 5 - 6\cos(\pi t)$

Amp: 6 Period: $\frac{2\pi}{\pi} = 2$

Freq: $\frac{1}{2}$

3. $h(\theta) = -24\sin(8\pi\theta)$

Amp: 24 Period: $\frac{2\pi}{8\pi} = \frac{1}{4}$

Freq: 4

Use the given information to create a *sine* function.

4.

Amplitude: 5
Period: 4π
Vertical Shift: down 4

$\frac{2\pi}{b} = 4\pi$

$\frac{b}{2\pi} = \frac{1}{4\pi}$

$b = \frac{1}{2}$

$y = 5\sin\left(\frac{x}{2}\right) - 4$

5.

Amplitude: 2
Period: $\frac{3\pi}{5}$
Vertical Shift: up 9

$\frac{2\pi}{b} = \frac{3\pi}{5}$

$\frac{b}{2\pi} = \frac{5}{3\pi}$

$b = \frac{10}{3}$

$y = 2\sin\left(\frac{10}{3}x\right) + 9$

6.

Amplitude: 1
Period: 4
Vertical Shift: up 1

$\frac{2\pi}{b} = 4$

$\frac{b}{2\pi} = \frac{1}{4}$

$b = \frac{\pi}{2}$

$y = \sin\left(\frac{\pi}{2}x\right) + 1$

Identify the listed information and graph the trig function.

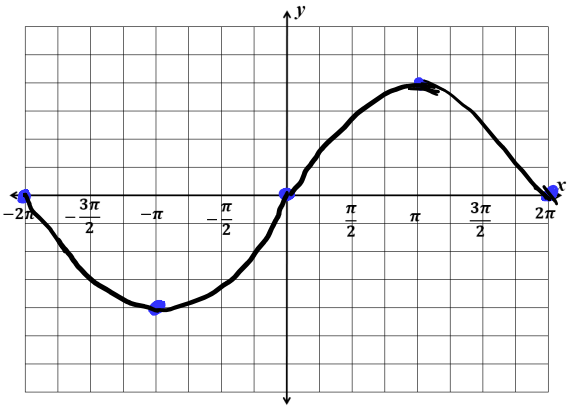
7. $y = 4\sin\left(\frac{x}{2}\right)$

$\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2$

Amp: 4 Period: 4π

Midline: $y=0$ Freq: $\frac{1}{4\pi}$

Max value: 4 Min value: -4



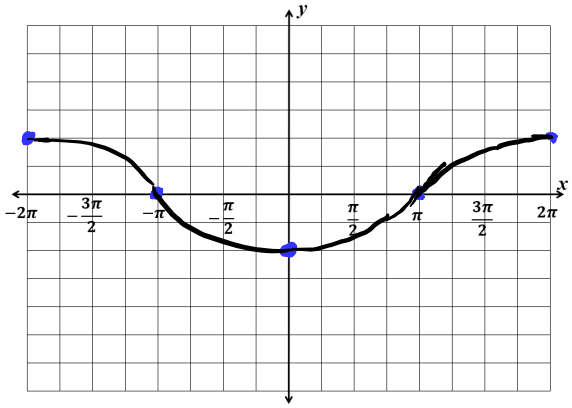
8. $y = -2\cos\frac{1}{2}\theta$

$\frac{2\pi}{\frac{1}{2}} = 2\pi \cdot 2$

Amp: 2 Period: 4π

Midline: $y=0$ Freq: $\frac{1}{4\pi}$

Max value: 2 Min value: -2



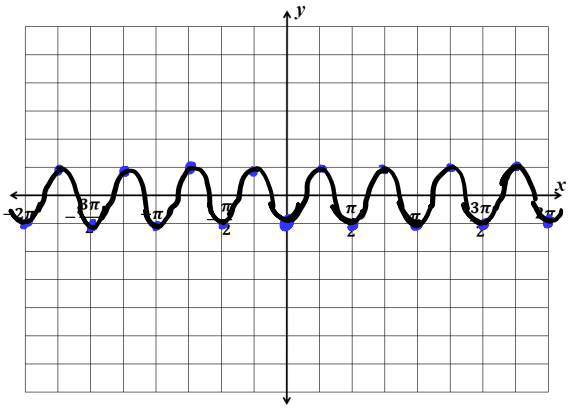
9. $y = -\cos 4x$

$\frac{2\pi}{4}$

Amp: 1 Period: $\frac{\pi}{2}$

Midline: $y=0$ Freq: $\frac{2}{\pi}$

Max value: 1 Min value: -1



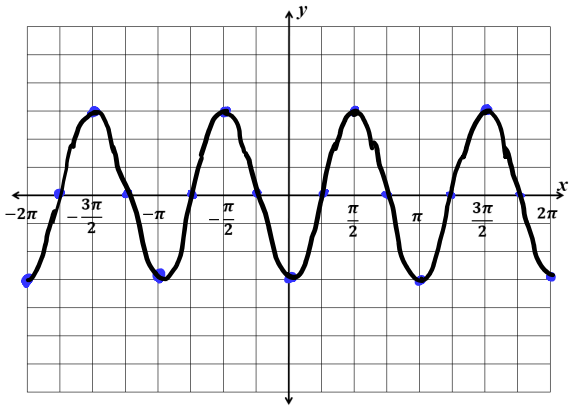
10. $y = -3\cos 2x$

$\frac{2\pi}{2}$

Amp: 3 Period: π

Midline: $y=0$ Freq: $\frac{1}{\pi}$

Max value: 3 Min value: -3

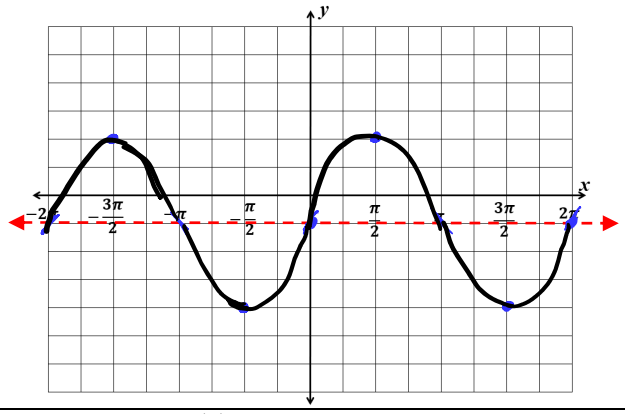


11. $y = 3\sin x - 1$

Amp: 3 Period: 2π

Midline: $y=-1$ Freq: $\frac{1}{2\pi}$

Max value: 2 Min value: -4

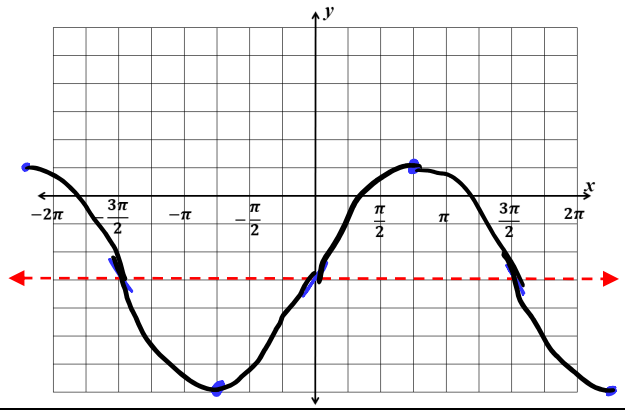


12. $y = -3 + 4\sin\frac{2}{3}x$ $\frac{2\pi}{2/3} = 2\pi \cdot \frac{3}{2}$

Amp: 4 Period: 3π

Midline: $y=-3$ Freq: $\frac{1}{3\pi}$

Max value: 1 Min value: -7

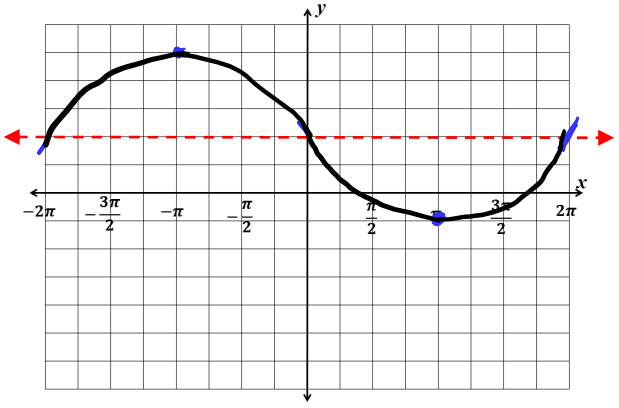


13. $y = 2 - 3\sin\left(\frac{x}{2}\right)$ $\frac{2\pi}{1/2} = 2\pi \cdot 2$

Amp: 3 Period: 4π

Midline: $y=2$ Freq: $\frac{1}{4\pi}$

Max value: 5 Min value: -1

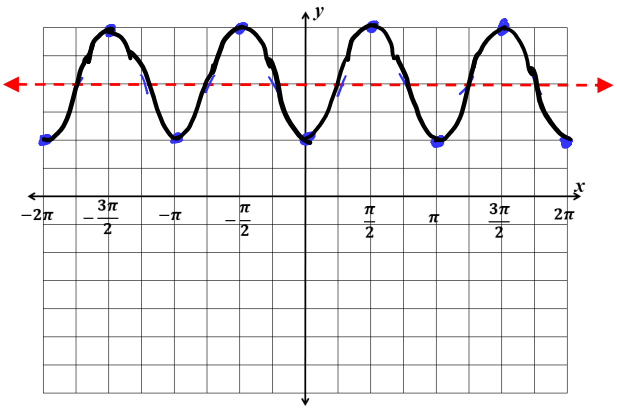


14. $y = -2\cos(2x) + 4$ $\frac{2\pi}{2}$

Amp: 2 Period: π

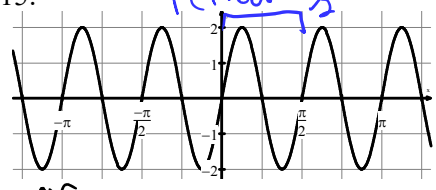
Midline: $y=4$ Freq: $\frac{1}{\pi}$

Max value: 6 Min value: 2



Write the equation of the following *sine* curves.

15. Period $= \frac{\pi}{2}$

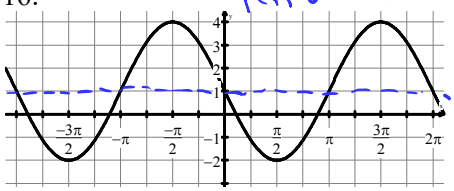


$\frac{2\pi}{b} = \frac{\pi}{2}$ $b=4$

$\frac{b}{2\pi} = \frac{2}{\pi}$

$y = 2\sin(4x)$

16. Period $= 2\pi$

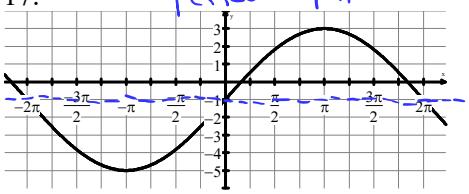


$a=-3$ $d=1$

$b=1$

$y = -3\sin(x) + 1$

17. Period $= 4\pi$



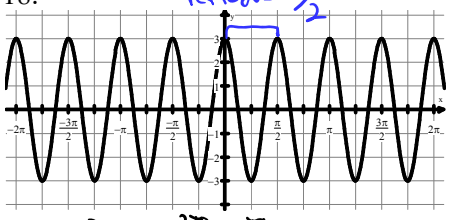
$a=4$ $\frac{2\pi}{b} = 4\pi$ $b=\frac{1}{2}$

$d=-1$ $\frac{b}{2\pi} = \frac{1}{4\pi}$

$y = 4\sin\left(\frac{x}{2}\right) - 1$

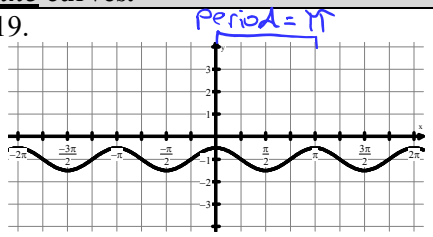
Write the equation of the following *cosine* curves.

18.



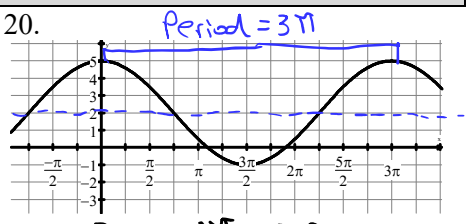
$y = 3\cos(4x)$

19.



$y = \frac{1}{2}\cos(2x) - 1$

20.



$y = 3\cos\left(\frac{2}{3}x\right) + 2$

3.6A Sinusoidal Function Transformations

3.6A Test Prep

21. **Calculator active.** The cycle between low tides and high tides at a beachfront town can be measured by

$$f(x) = a\sin(bx) + d$$

where a, b, and k are constants. The maximum value of the tide is 12.04 feet, and the minimum value of the tide is 1.36 feet. Assuming we know the values of b and d, which of the following would best define $f(x)$?

(A) $10.68\sin(bx) + d$

(B) $6.7\sin(bx) + d$

(C) $6.02\sin(bx) + d$

(D) $5.34\sin(bx) + d$

D

The amplitude is half the difference between the maximum and minimum values.

$$\frac{12.04 - 1.36}{2} = 5.34$$

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22. **Calculator active.** The table gives minimum temperature, in degrees Fahrenheit, on the first day of each of 8 months in a certain city. The function f given by $f(\theta) = a\sin(b\theta) + d$, where a, b, and d are constants, is used to model these data with θ representing the number of the month. Assume the period of f is 12 months.

Month	1	2	3	4	5	6	7	8
Temperature (degrees Fahrenheit)	28.6	23.2	34.9	40.3	48.6	56.7	61.0	68.2

Based on the data in the table, which of the following is the best value for d?

D

(A) $\frac{\pi}{6}$ (B) 22.5 (C) 45 (D) 45.7

The midline is determined by finding the average of the maximum and minimum values. $\frac{23.2+68.2}{2} = 45.7$

23. The table gives ordered pairs for seven points from a larger data set. The larger data set can be modeled by a sinusoidal function f with a period of 6. The minimum values of the data set occur at x-values that are multiples of 6.

x	0	1	2	3	4	5	6
$f(x)$	-4	-1	3	6	3	-1	-4

Which of the following best defines $f(x)$ for the larger data set?

(A) $-4\cos(12\pi x)+1$

(B) $-4\cos\left(\frac{\pi}{3}x\right)+1$

(C) $-5\cos(12\pi x)+1$

(D) $-5\cos\left(\frac{\pi}{3}x\right)+1$

D

$\frac{2\pi}{b}=6$

$\frac{2\pi}{6}=b$

$\frac{\pi}{3}=b$

$a=\frac{\text{Max}-\text{Min}}{2}=\frac{6--4}{2}=5$