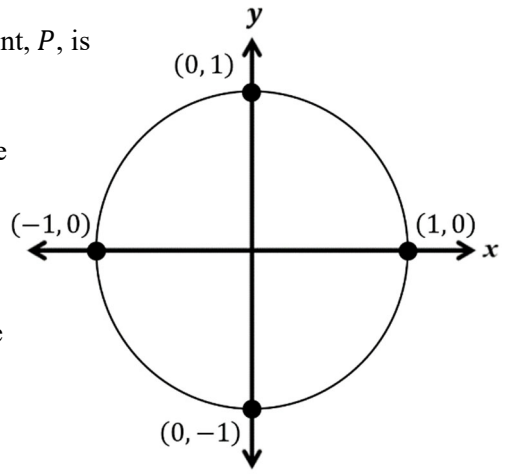
AP Precalc

Write your questions and thoughts here!

3.8 The Tangent Function

3.8 Notes

Take a unit circle and form an angle in standard position. The point, P, is the intersection of the terminal ray and the circle.



The tangent function, $f(\theta) = \tan\theta$, gives the _____ of the terminal ray.

The slope of the terminal ray can also be described by the ratio of the change in y-values to the change in x-values between any two points on the ray. The tangent function is also the ratio of the sine function to the cosine function. Therefore

$$\tan\theta =$$

as long as $\cos\theta \neq 0$.

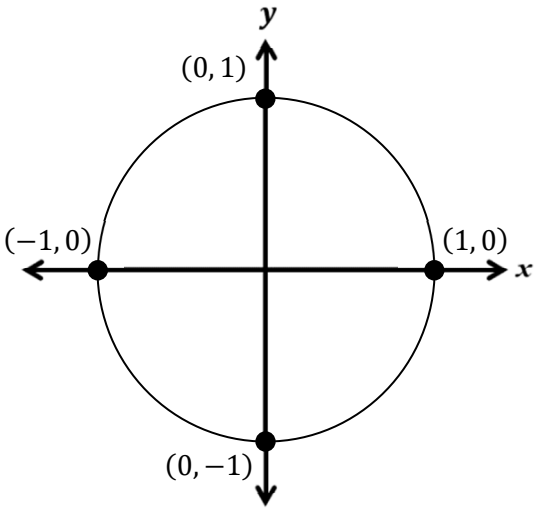
1. In the xy-plane, an angle θ, in standard position, has a measure of $\theta = \frac{\pi}{3}$. What is the slope of the terminal ray of the angle?

Finding the tangent values is the same as finding the slope of the terminal ray of the angle.

Evaluate.		
2. $\tan\frac{\pi}{4}$	3. $\tan\frac{5\pi}{6}$	4. $\tan\frac{3\pi}{2}$

Think about the slope values of the terminal ray as it moves around the unit circle.

- Slope starts as _____.
- Gets larger until it approaches _____.
- At $\theta = \frac{\pi}{2}$, the slope is _____.
- Then the slope is very negative but starts to grow towards zero.
- Once we reach an angle of ___, we are back to a slope of zero again.
- As soon as we pass the angle π, the slope will be the same as it was in _____.
- Every _____ revolution of the circle, the tangent function repeats.

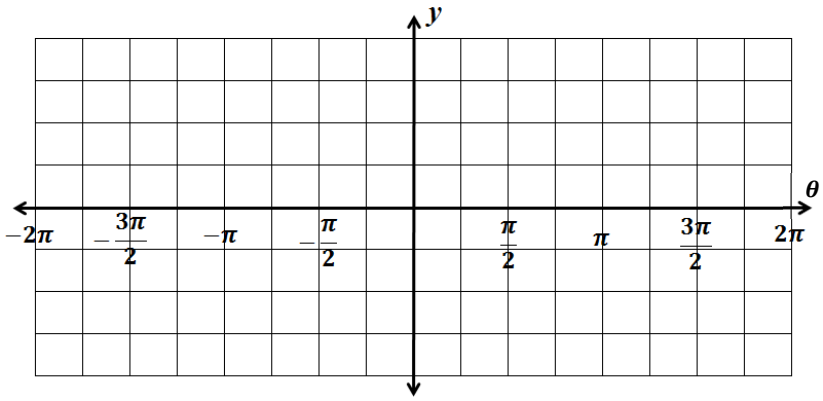


The period of $\tan\theta$ is ____.

Write your questions and thoughts here!

Each time that $\cos\theta = 0$, $\tan\theta$ is undefined. What angle on the unit circle does this occur?

For the graph of $f(\theta) = \tan\theta$, this is represented by vertical asymptotes.



Vertical Asymptotes

For the graph of $f(\theta) = \tan\theta$, a vertical asymptote appears at every $\theta = \frac{\pi}{2} + k\pi$, for integer values of k.

For the graph of $\tan(b\theta)$, the period is

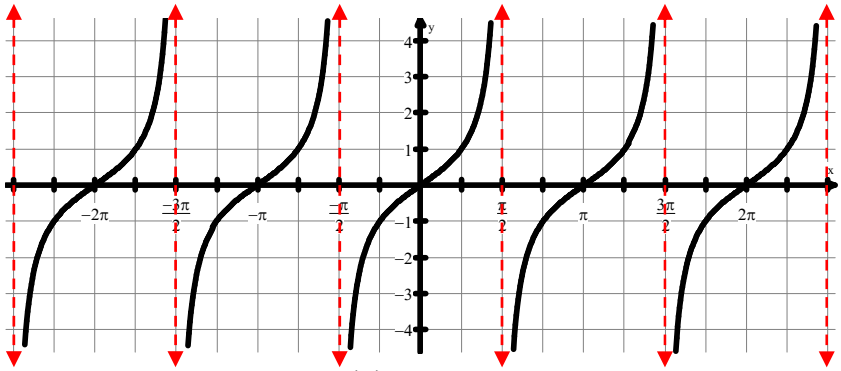
A vertical asymptote appears at every $\theta = \frac{\pi}{\quad} + k\frac{\pi}{\quad}$, for integer values of k.

Write an equation that represents all asymptotes of the graph of f in the xy-plane.

5. $f(\theta) = \tan(2\theta)$

6. $f(\theta) = \tan\left(\frac{2\theta}{3}\right)$

Characteristics of the tangent graph



$f(\theta) = \tan\theta$

- The tangent function increases.
- Its graph changes from concave down to concave up between each set of asymptotes.
- The point where it changes concavity is called an inflection point.

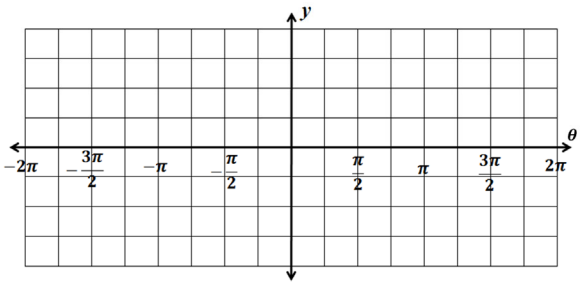
Write your questions and thoughts here!

The graph of $g(\theta) = a\tan\big(b(\theta + c)\big) + d$ is transformation of the graph of $f(\theta) = \tan\theta$ in the following ways:

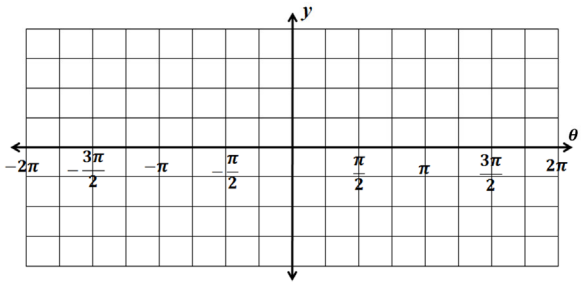
- **The constant a** creates a _____ by a factor of $|a|$. If $a < 0$, there is a reflection over the _____.
- **The constant b** creates a _____ and changes the period by a factor of $\left|\frac{1}{b}\right|$. If $b < 0$, there is a reflection over the _____.
- **The constant c** creates a _____ (phase shift) by $-c$ units.
- **The constant d** creates a _____ by d units.

Graph each function.

7. $y = \frac{1}{2}\tan\left(2\theta - \frac{\pi}{2}\right) + 1$



8. $y = -3\tan\left(\frac{\theta}{2} + \frac{\pi}{2}\right) - 2$



3.8 The Tangent Function

AP Precalculus

3.8 Practice

Write an equation that represents all asymptotes of the graph of f in the xy-plane.

1. $f(\theta) = \tan(3\theta)$

2. $f(\theta) = \tan(6\theta)$

3. $f(\theta) = \tan\left(\frac{\theta}{5}\right)$

In the xy-plane, the angle θ is in standard position. What is the slope of the terminal ray of the angle?

4. $\theta = \frac{\pi}{6}$

5. $\theta = \frac{3\pi}{4}$

6. $\theta = \frac{5\pi}{3}$

Evaluate.

7. $\tan\frac{\pi}{2}$

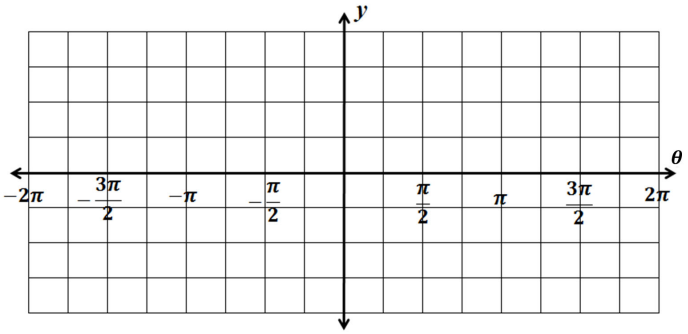
8. $\tan\frac{4\pi}{3}$

9. $\tan\frac{5\pi}{4}$

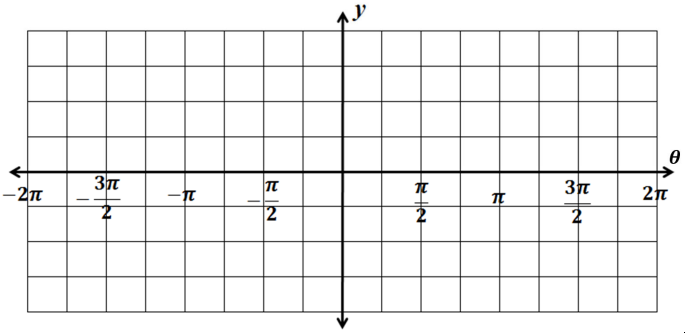
10. $\tan\frac{11\pi}{6}$

Graph each trig function.

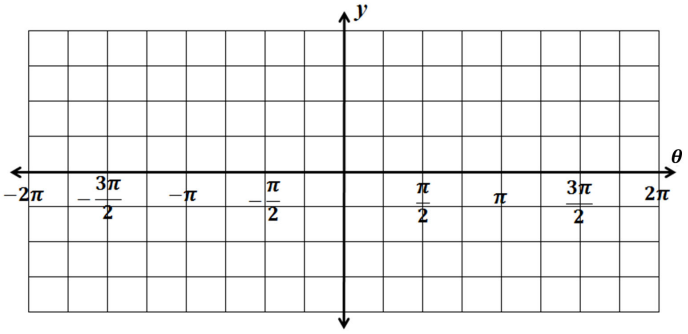
11. $y = -\frac{1}{2}\tan\theta$



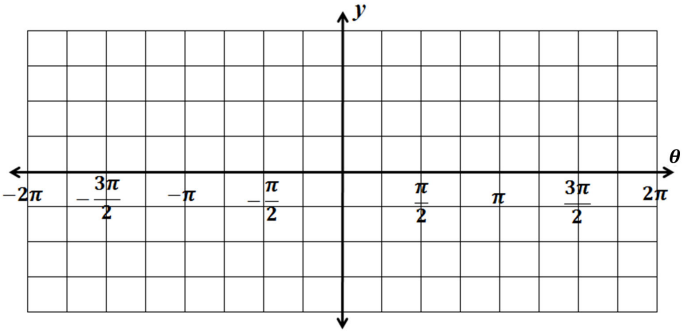
12. $y = 4\tan\left(\frac{1}{2}\theta\right) - 3$



13. $y = 2\tan\left(\theta - \frac{\pi}{4}\right) - 1$



14. $y = \tan(2\theta + \pi) + 2$



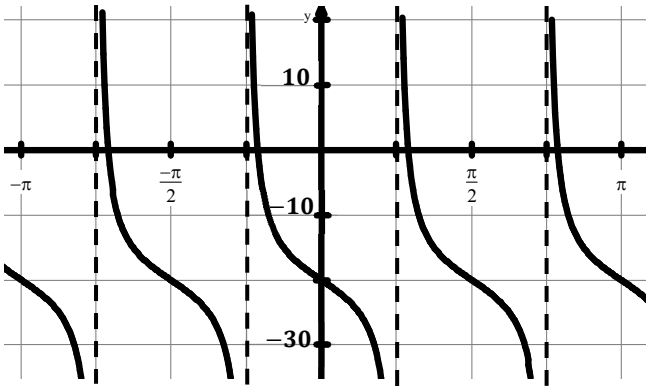
3.8 The Tangent Function

3.8 Test Prep

15. The graph of the function g is given in the xy-plane. If $g(x) = a\tan(bx) - 20$, where a and b are constants, which of the following could be true?

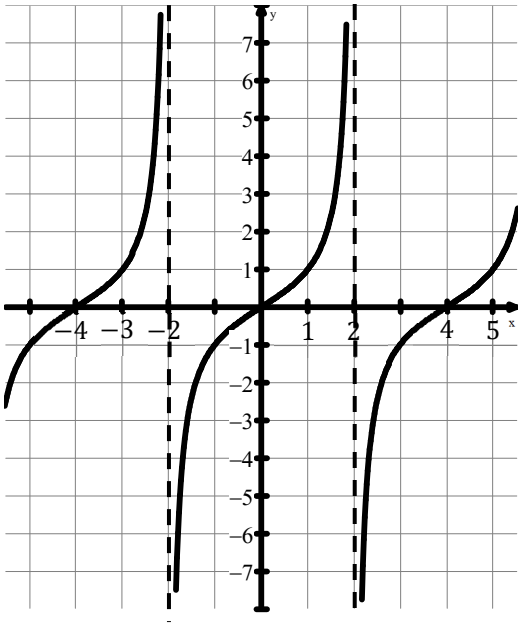
I. If $a > 0$ then $b > 1$
II. If $a > 0$ then $b < 0$
III. If $a < 0$ then $b > 1$
IV. If $a < 0$ then $b < 0$

(A) III only
(B) IV only
(C) I and IV only
(D) II and III only



Graph of g

16. The graph of $f(x) = \tan(bx)$, where b is a constant, is shown in the xy-plane. What is the value of b?



Graph of f

(A) 4

(B) 2

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

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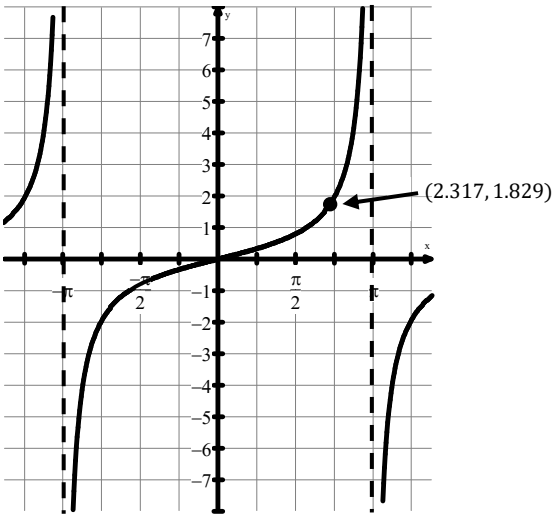
17. The figures shows the graph of $f(x) = a\tan(bx)$, where a and b are constants, in the xy-plane. The graph of f has two vertical asymptotes at $x = -\pi$ and $x = \pi$, and a point with coordinates given is on the graph of f. What are all solutions to $f(x) = 1.829$?



Graph of f

(A) $x = 2.317$ only

(B) $x = 2.317 + \pi k$, where k is any integer

(C) $x = 2.317 + 2\pi k$, where k is any integer

(D) $x = 2.317 + 3\pi k$, where k is any integer