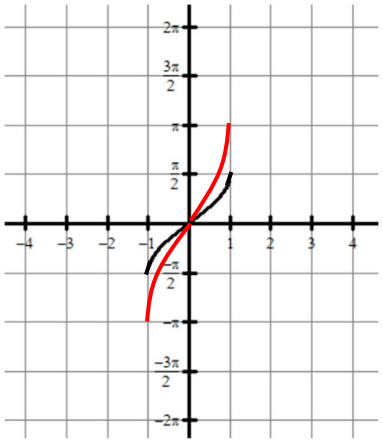
3.9 Inverse Trigonometric Functions

AP Precalculus

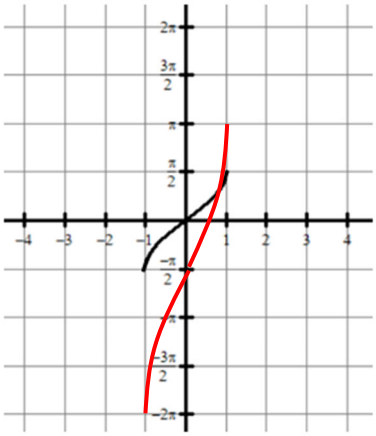
3.9 Practice

The graph of $f(x) = sin^{-1}(x)$ is shown below. Use the graph of f to graph $g(x)$.

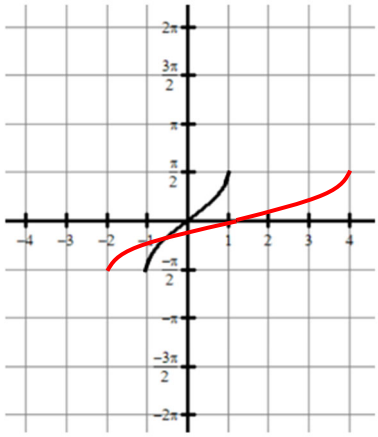
1. $g(x) = 2\sin^{-1}(x)$



2. $g(x) = 3\sin^{-1}(x) - \frac{\pi}{2}$

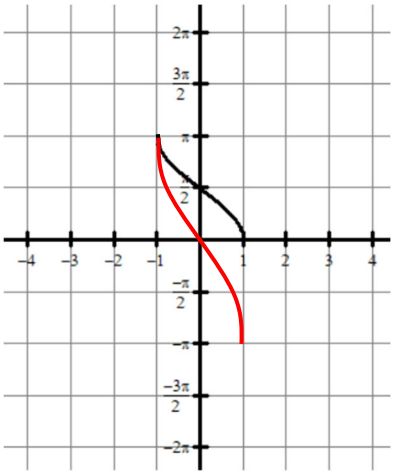


3. $g(x) = \sin^{-1}\left(\frac{1}{3}(x-1)\right)$

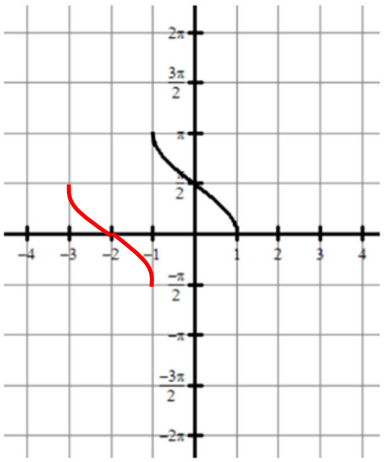


The graph of $f(x) = cos^{-1}(x)$ is shown below. Use the graph of f to graph $g(x)$.

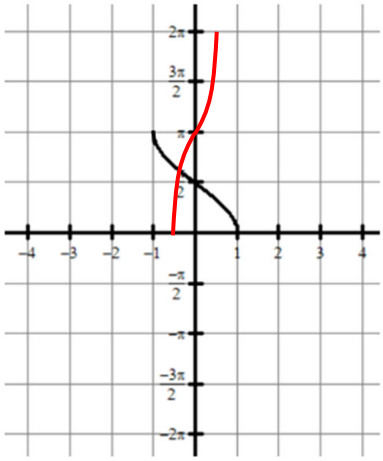
4. $g(x) = 2\cos^{-1}(x) - \pi$



5. $g(x) = \cos^{-1}(x+2) - \frac{\pi}{2}$

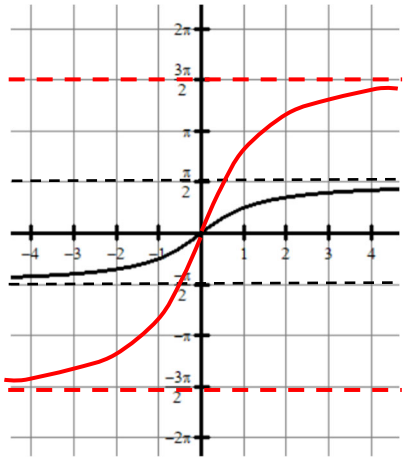


6. $g(x) = 2\cos^{-1}(-2x)$

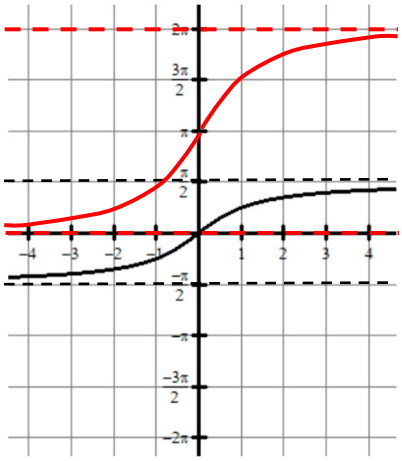


The graph of $f(x) = tan^{-1}(x)$ is shown below. Use the graph of f to graph $g(x)$.

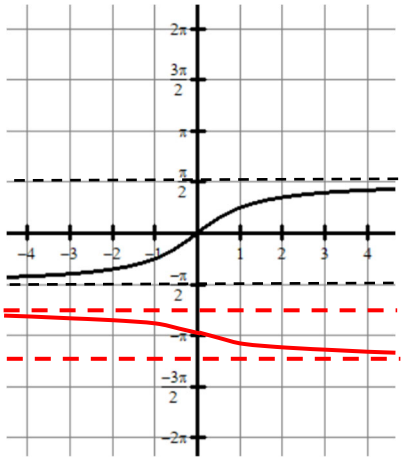
7. $g(x) = 3\tan^{-1}(x)$



8. $g(x) = 2\tan^{-1}(x) + \pi$



9. $g(x) = \frac{1}{2}\tan^{-1}(-x) - \pi$



Find the inverse of each function and list the domain and range of $f^{-1}(x)$.

10. $f(x) = 5\sin x - 3$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$f^{-1}(x)$

$x = 5\sin y - 3$

$+3 \quad +3$

$\frac{x+3}{5} = \frac{5\sin y}{5}$

$\frac{x+3}{5} = \sin y$

$\sin^{-1}\left(\frac{x+3}{5}\right) = \sin^{-1}(\sin y)$

$\sin^{-1}\left(\frac{x+3}{5}\right) = y$

$f^{-1}(x) = \sin^{-1}\left(\frac{x+3}{5}\right)$

Domain: $-8 \le x \le 2$

Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Range of $f(x)$:

$5\sin\left(-\frac{\pi}{2}\right) - 3$

$5(-1) - 3$

-8

$5\sin\left(\frac{\pi}{2}\right) - 3$

$5(1) - 3$

2

$-8 \le y \le 2$

is domain of $f^{-1}(x)$

11. $f(\theta) = \frac{1}{2}\tan\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$f^{-1}(x)$

$x = \frac{1}{2}\tan y$

$2 \cdot x = 2 \cdot \frac{1}{2}\tan y$

$2x = \tan y$

$\tan^{-1}(2x) = \tan^{-1}(\tan y)$

$\tan^{-1}(2x) = y$

$f^{-1}(\theta) = \tan^{-1}(2\theta)$

Domain: All real numbers

Range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Range of $f(x)$:

$\frac{1}{2}\tan\left(-\frac{\pi}{2}\right)$

$\frac{1}{2}\left(\frac{-1}{0}\right)$ = undefined

$\frac{1}{2}\tan\left(\frac{\pi}{2}\right)$

$\frac{1}{2}\left(\frac{1}{0}\right)$ = undefined

all real numbers

is domain of $f^{-1}(x)$

12. $f(x) = 4 - \cos(2x)$ for $0 \le x \le \frac{\pi}{2}$

$f^{-1}(x)$

$x = 4 - \cos(2y)$

$-4 \quad -4$

$\frac{x-4}{-1} = \frac{-\cos(2y)}{-1}$

$-x + 4 = \cos(2y)$

$\cos^{-1}(-x+4) = \cos^{-1}(\cos(2y))$

$\frac{\cos^{-1}(-x+4)}{2} = \frac{2y}{2}$

$\frac{1}{2}\left[\cos^{-1}(-x+4)\right] = y$

$f^{-1}(x) = \frac{1}{2}\cos^{-1}(-x+4)$

Domain: $3 \le x \le 5$

Range: $0 \le y \le \frac{\pi}{2}$

Range of $f(x)$:

$4 - \cos(2 \cdot 0)$

$4 - 1$

3

$4 - \cos\left(2 \cdot \frac{\pi}{2}\right)$

$4 - (-1)$

5

$3 \le y \le 5$

is domain of $f^{-1}(x)$

13. $f(\theta) = 3\sin(\theta - \pi) + 1$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$f^{-1}(x)$

$x = 3\sin(y - \pi) + 1$

$-1 \quad -1$

$\frac{x-1}{3} = \frac{3\sin(y-\pi)}{3}$

$\frac{x-1}{3} = \sin(y - \pi)$

$\sin^{-1}\left(\frac{x-1}{3}\right) = \sin^{-1}(\sin(y-\pi))$

$\sin^{-1}\left(\frac{x-1}{3}\right) = y - \pi$

$+\pi \quad +\pi$

$\sin^{-1}\left(\frac{x-1}{3}\right) + \pi = y$

$f^{-1}(\theta) = \sin^{-1}\left(\frac{\theta-1}{3}\right) + \pi$

Domain: $-2 \le x \le 4$

Range: $\frac{\pi}{2} \le y \le \frac{3\pi}{2}$

Range of $f(x)$:

$3\sin\left(-\frac{\pi}{2} - \pi\right) + 1$

$3\sin\left(-\frac{3\pi}{2}\right) + 1$

$3(1) + 1 = 4$

$3\sin\left(\frac{\pi}{2} - \pi\right) + 1$

$3\sin\left(-\frac{\pi}{2}\right) + 1$

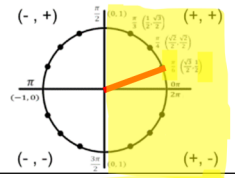
$3(-1) + 1 = -2$

$-2 \le y \le 4$

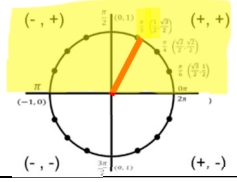
is domain of $f^{-1}(x)$

Evaluate the following expressions. Find the principal value in radians. NO CALCULATOR!!

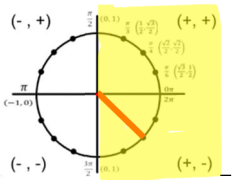
14. $\sin^{-1}\left(\frac{1}{2}\right)$ $\quad \frac{\pi}{6}$



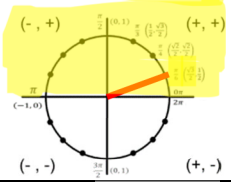
15. $\cos^{-1}\left(\frac{1}{2}\right)$ $\quad \frac{\pi}{3}$



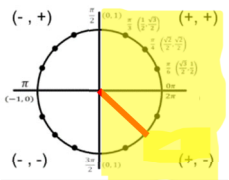
16. $\tan^{-1}(-1)$ $\quad -\frac{\pi}{4}$



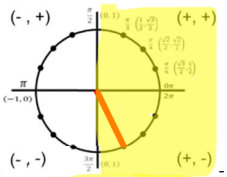
17. $\arccos\left(\frac{\sqrt{3}}{2}\right)$ $\quad \frac{\pi}{6}$



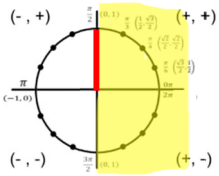
18. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)$ $\quad -\frac{\pi}{4}$



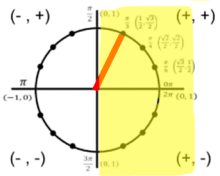
19. $\tan^{-1}(-\sqrt{3})$ $\quad -\frac{\pi}{3}$



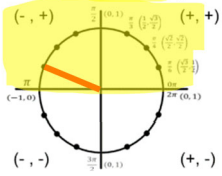
20. $\sin^{-1}(1)$ $\quad \frac{\pi}{2}$



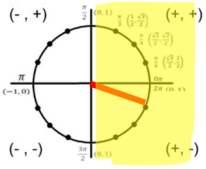
21. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ $\quad \frac{\pi}{3}$



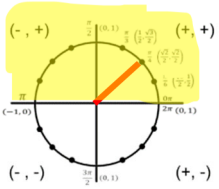
22. $\arccos\left(-\frac{\sqrt{3}}{2}\right)$ $\quad \frac{5\pi}{6}$



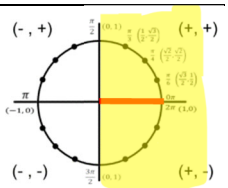
23. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$ $\quad -\frac{\pi}{6}$



24. $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\quad \frac{\pi}{4}$



25. $\arcsin(0)$ $\quad 0\pi$

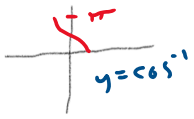


3.9 Inverse Trigonometric Functions

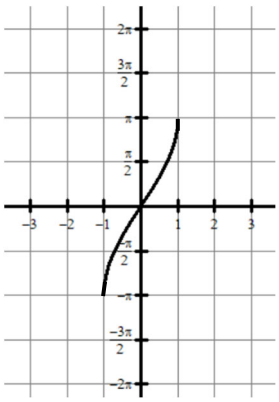
3.9 Test Prep

26. Which of the following is the graph of $f(x) = 2\cos^{-1}(x)$?

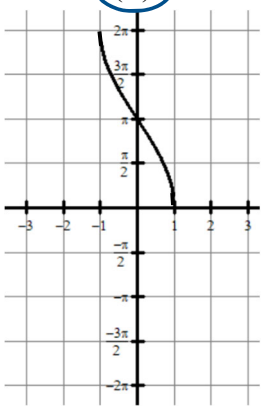
vertical dilation of 2



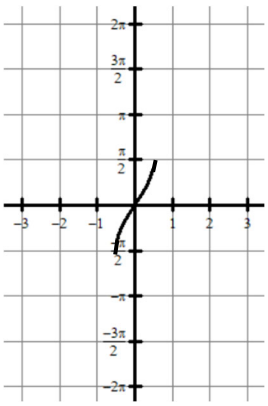
(A)



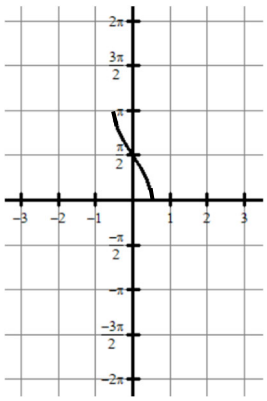
(B)



(C)



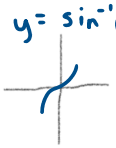
(D)



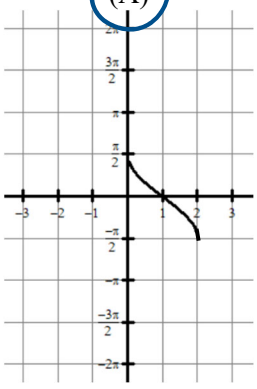
27. Given $f(x) = 1 - \sin(x)$, which of the following is the graph of $f^{-1}(x)$?

$f^{-1}(x) = \sin^{-1}[-(x-1)]$

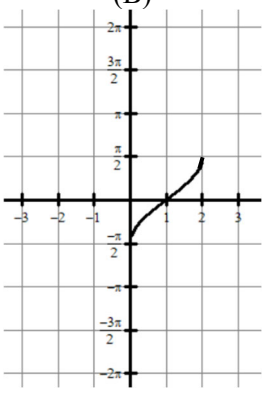
horizontal reflection
translate right 1



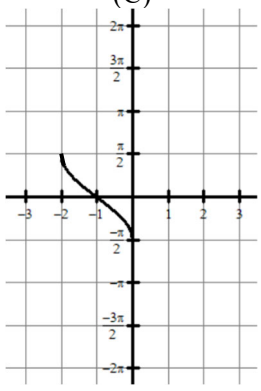
(A)



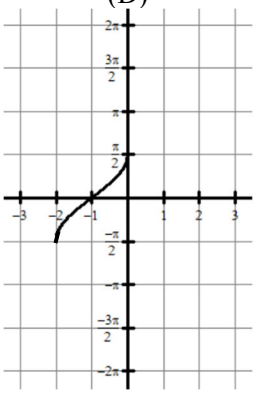
(B)



(C)



(D)



28. Given $f(x) = 2\sin\left(\frac{1}{3}x\right) + 1$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, what is the domain of $f^{-1}(x)$?

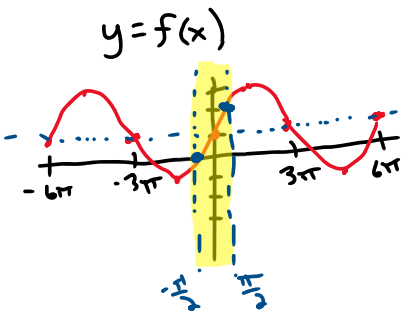
(A) $\left[-\frac{3\pi}{2}, \frac{3\pi}{2}\right]$

(B) $[0, 2]$

(C) $[-1, 3]$

(D) $[-\pi, \pi]$

The range of $f(x)$ is the domain of $f^{-1}(x)$



Min

$f(-\frac{\pi}{2}) = 2\sin\left(\frac{1}{3}(-\frac{\pi}{2})\right) + 1$

$2\sin(-\frac{\pi}{6}) + 1$

$2(-\frac{1}{2}) + 1$

$-1 + 1$

$f(-\frac{\pi}{2}) = 0$

Max

$f(\frac{\pi}{2}) = 2\sin\left(\frac{1}{3}(\frac{\pi}{2})\right) + 1$

$2\sin(\frac{\pi}{6}) + 1$

$2(\frac{1}{2}) + 1$

$1 + 1$

$f(\frac{\pi}{2}) = 2$

range of $f(x)$ is $[0, 2]$