4.1 Parametric Functions

Write your questions and thoughts here!

For most of algebra, we have become accustomed to equations with one dependent variable (y) and one independent variable (x). These are called *rectangular* equations and can look like this…

$$y = \frac{2}{3}x + 1 \qquad y = 4 - \frac{1}{2}x^2 \qquad y = \cos(2x)$$

Today, we're going to look at functions that have **two dependent variables** (x and y) and one independent varible (t). These are called *parametric* functions.

Parametric Function

Think of a parametric function as a function that has an equation for the x-coordinate, and a separate equation for the y-coordinate.

$$\boldsymbol{f(t) =}$$

We call t the ________________. At any time t, we know the coordinate point of $f(t)$ by substituting values of t into each equation $x(t)$ and $y(t)$.

1. At time $t = -1$, where is the parametric function $f(t) = (6 - t^2,\ t + 3)$?

Creating a table of numerical values

2. Create a numerical table for the function $f(t) = (3t, t^2 - 1)$

t	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
x									
y									

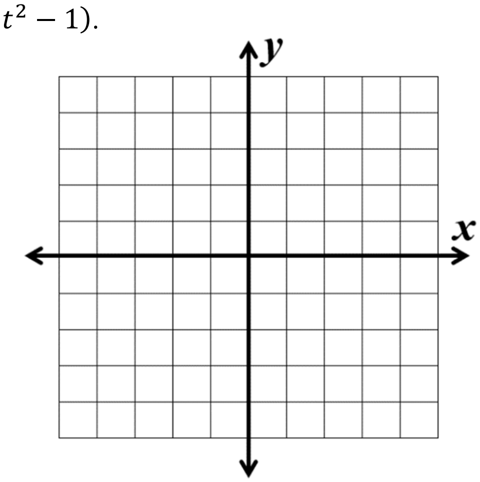
Graphing Parametric Equations

Let's look at the graph of the parametric function $f(t) = (3t, t^2 - 1)$.

a. Use the table of values and plot the points in the order from the lowest value of t to the highest value of t.

b. Connect the coordinate points **in order**.

c. Use arrows to indicate the direction of motion.



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This graph looks like a parabola, but we have more information now. Instead of just finding coordinate points, we know WHEN the graph was at each point and which direction it was moving.

There was **no restriction** on the domain $f(t) = (3t, t^2 - 1)$. Or in other words, there was no restriction on time (the parameter), so this graph will continue on both ends.

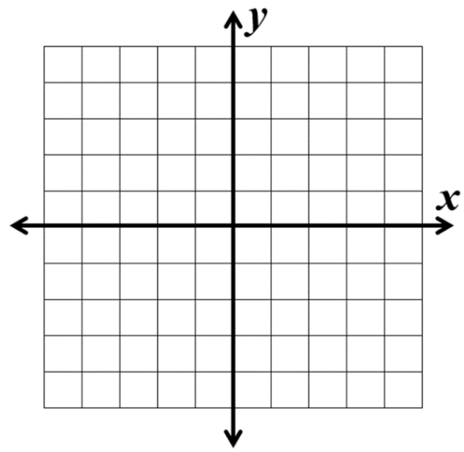
Restricted Domain

For a restricted domain, we restrict the parameter (usually time). You have a starting point and an ending point for the motion along the path given.

3. Use the table of values to sketch the parametric curve.

$$f(t) = \left(t^2 - 4, \frac{3t}{2}\right) \text{ for } -2 \le t \le 3$$

t	-2	-1	0	1	2	3
x	0	-3	-4	-3	0	5
y	-3	-1.5	0	1.5	3	4.5



The graph does not continue to infinity because the parameter is restricted.

- The beginning of the graph is at the point ___________.
- The end of the graph is at the point ___________.

4.1 Parametric Functions

AP Precalculus

4.1 Practice

Find $x(t)$ and $y(t)$ for each parametric function.

1. Let $f(t) = (2t - 1, \ln 2t)$.
   a. What is $x(t)$?
   b. What is $y(t)$?

2. Let $f(t) = (e^{3t}, \sin(t))$.
   a. What is $x(t)$?
   b. What is $y(t)$?

Find the coordinate point of the parametric function at the given value of the parameter.

3. At time $t = 9$, where is the parametric function $f(t) = \left(\frac{t-2}{t+1}, \sqrt{t-5}\right)$?

4. At time $t = -2$, where is the parametric function $f(t) = \left(t^2 + 3, \frac{1}{t}\right)$?

5. At time $t = 3$, where is the parametric function $f(t) = (t + 6, 5 - t^2)$?

6. Given the parametric function $f(\theta) = (3\cos\theta, 3\sin\theta)$, complete the table of numerical values for the given values of θ. No calculator.

$\boldsymbol{\theta}$	$-\frac{\pi}{2}$	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$\boldsymbol{x}$					
$\boldsymbol{y}$					

7. **Calculator active.** Complete the table for the given parametric function, $f(t) = (e^t, t^2 - 1)$.

$\boldsymbol{t}$	-1	-0.75	-0.5	-0.25	0
$\boldsymbol{x}$					
$\boldsymbol{y}$					

8. Which parametric function could be used to construct the given table of numerical values?

$\boldsymbol{t}$	-2	-1	0	1	2
$\boldsymbol{x}$	7	4	1	-2	-5
$\boldsymbol{y}$	-1	1	3	5	7

(A) $(-3t + 1, 2t - 3)$

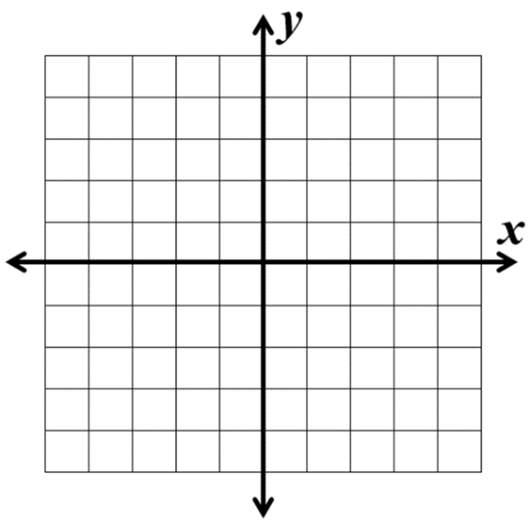
(B) $(1 + 3t, 2t - 3)$

(C) $(1 - 3t, 2t + 3)$

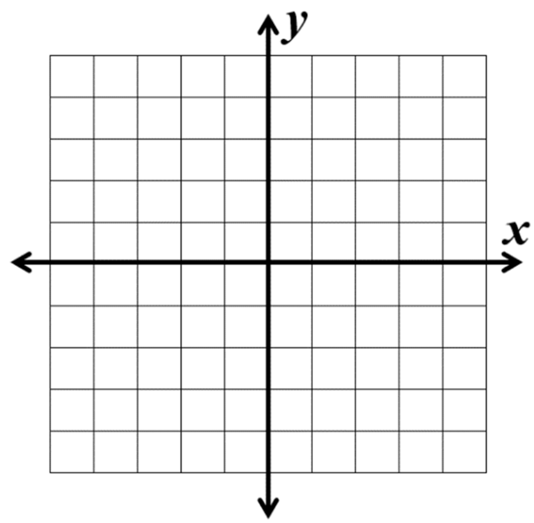
(D) $(1 - 2t, 3t + 2)$

Sketch the curve represented by each parametric function. Check your answer with the use of technology.

9. $f(t) = \left(\frac{t^2}{2}, t-2\right), -3 \leq t \leq 3$



10. $f(t) = (t+2, 1-t), -2 \leq t \leq 3$



Find the domain of each parametric function.

11. $f(t) = \left(\sqrt{1-t}, t^3 - 1\right)$

12. $f(t) = \left(\frac{5}{3t}, \sqrt{t+2}\right)$