AP Precalc

### 4.2 Parametric Functions Modeling Planar Motion

### 4.2 Notes

A parametric function given by $f(t)=(x(t), y(t))$ can be used to model particle motion in the plane. Remember, a rectangular equation tells us where the object has been, but with the addition of the third variable (often $t$, to represent time), a parametric equation tells us when the object was at a point $(x, y)$.

1. A parametric function is given by $f(t)=\left(t^{3}-4 t, 3 t\right)$ with a restricted domain of $-2 \leq t \leq 2$. Use the table below to sketch the curve and indicate the direction of motion using arrows.

| $\boldsymbol{t}$ | -2 | -1 | 0 | 1 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{x}$ | 0 | 3 | 0 | -3 | 0 |
| $\boldsymbol{y}$ | -6 | -3 | 0 | 3 | 6 |

If there was no restriction on the domain, what would be the position of the particle at $t=10$ ?

CALCULATOR PRACTICE!


## Horizontal and Vertical Extrema

Let $f(t)=(x(t), y(t))$ be a parametric function.

- The horizontal extrema of $f(t)$ occur at the maximum and minimum values of $\qquad$
- The vertical extrema of $f(t)$ occur at the maximum and minimum values of $\qquad$ .

2. Let $f(t)=\left(1-t, t^{2}-1\right)$ for $-2 \leq t \leq 5$.
a. Determine the horizontal extrema.
b. Determine the vertical extrema.

On the next page, we can look at a graph of the function to verify our results.

Notice the endpoints don't have arrows, because of the domain restriction!

What if we are not given a domain restriction? Then we would need to analyze the functions $x(t)$ and $y(t)$ to see if they present any domain restrictions on $t$. If $x(t)$ is linear, then there would be no max or min value of $x$ on the curve. The same would hold true for $y(t)$ and the $\max$ or $\min y$ value.


## Examples with no domain restrictions (end points)

3. $f(t)=\left(t-1, \frac{t}{2}+1\right)$
a. Determine the horizontal extrema.
b. Determine the vertical extrema.

4. $f(t)=(t+3, \sqrt{t+1})$
a. Determine the horizontal extrema.
b. Determine the vertical extrema.


## $\boldsymbol{X}$-intercepts and $\boldsymbol{Y}$-intercepts of a Parametric Function

Let $f(t)=(x(t), y(t))$ be a parametric function.

- The $\boldsymbol{x}$-intercept occurs at $t$ values when $\qquad$ .
- The $\boldsymbol{y}$-intercept occurs at $t$ values when $\qquad$ .

5. Find the $x$ - and $y$-intercepts of the function $f(t)=\left(t^{2}+5 t+6, t+4\right)$. Verify these results by using technology to look at the graph.

For each parametric function, answer each part that follows. A graphing calculator should only be used to check your answers.

1. $f(x)=\left(\frac{1}{2} t, t^{2}-1\right)$ for $-3 \leq t \leq 1$
a. Graph the curve represented by the given parametric function. Indicate the direction of movement of the particle on your graph.

b. Find the horizontal relative extrema.
c. Find the vertical relative extrema.
d. Find the $x$-intercept(s). Show your work.
e. Find the $y$-intercept(s). Show your work.
2. $f(x)=(|t-1|, t+2)$ for $-4 \leq t \leq 4$
a. Graph the curve represented by the given parametric function. Indicate the direction of movement of the particle on your graph.

b. Find the horizontal relative extrema.
c. Find the vertical relative extrema.
d. Find the $x$-intercept(s). Show your work.
e. Find the $y$-intercept(s). Show your work.
3. $f(x)=\left(-(t+1)^{2}+3, t+1\right)$ for $-3 \leq t \leq 2$
a. Graph the curve represented by the given parametric function. Indicate the direction of movement of the particle on your graph.

b. Find the horizontal relative extrema.
c. Find the vertical relative extrema.
d. Find the $x$-intercept(s). Show your work.
e. Find the $y$-intercept(s). Show your work.

## Calculator active.

4. $f(x)=\left(t^{3}, t^{2}\right)$ for $-2 \leq t \leq 2$
a. Graph the curve represented by the given parametric function. Indicate the direction of movement of the particle on your graph.

b. Find the horizontal relative extrema.
c. Find the vertical relative extrema.
d. Find the $x$-intercept(s). Show your work.
e. Find the $y$-intercept(s). Show your work.

## For each parametric function, find the $\boldsymbol{x}$ - and $\boldsymbol{y}$-intercepts algebraically.

5. $f(t)=\left(t^{2}, t^{4}-1\right)$
a. $x$-intercept(s).
b. $y$-intercept(s).
6. $f(t)=\left(\ln (2 t), t^{2}\right)$
a. $x$-intercept(s).
b. $y$-intercept(s).
7. $f(t)=\left(-(t+2)^{2}+4, t+1\right)$
a. $x$-intercept(s).
b. $y$-intercept(s).

## Find the horizontal and vertical extrema of each parametric function.

8. $x(t)=2 t+1, y(t)=3-2 t$
a. Find the horizontal extrema.
9. $x(t)=t, y(t)=4$
a. Find the horizontal extrema.
b. Find the vertical extrema.
b. Find the vertical extrema.
