4.3 Parametric Functions and Rates of Change

AP Precalculus

Solutions

4.3 Practice

The table below is a numerical table of values that describe the location of a particle over the interval $0 \le t \le 8$.

t	0	1	2	3	4	5	6	7	8
x	2	3	4	5	6	7	8	9	10
y	30	16	6	0	−2	0	6	16	30

Use the table to answer questions 1-5.

1. Find the average rate of change of $x(t)$ over the interval $1 \le t \le 4$.

$$\frac{x(4)-x(1)}{4-1} = \frac{6-3}{3} = \boxed{1}$$

2. Find the average rate of change of $y(t)$ over the interval $6 \le t \le 8$.

$$\frac{y(8)-y(6)}{8-6} = \frac{30-6}{2} = \frac{24}{2} = \boxed{12}$$

3. Over which interval is the direction of particle motion to the right and upward? Assume the direction does not change between the values t shown in the table.

right: $0 \le t \le 8$

up: $4 \le t \le 8$

right and up: $\boxed{4 \le t \le 8}$

4. Over which interval is the direction of particle motion to the right and downward? Assume the direction does not change between the values t shown in the table.

right: $0 \le t \le 8$

down: $0 \le t \le 4$

right and down: $\boxed{0 \le t \le 4}$

5. Find the slope of the graph between the points that correspond to $t = 4$ and $t = 7$.

$$\frac{\frac{y(7)-y(4)}{7-4}}{\frac{x(7)-x(4)}{7-4}} = \frac{\frac{16-{-2}}{3}}{\frac{9-6}{3}} = \frac{\frac{18}{3}}{\frac{3}{3}} = \boxed{6}$$

Find a set of parametric equations that will produce the same path as the given set of equations with a direction of particle motion in the opposite direction. Use graphing technology to verify your answer.

6. $x(t) = -2(t+2)^2 + 4$, $y(t) = 3t - 4$ for $-3 \le t \le 1$.

$x(-t) = -2(2-t)^2 + 4$

$y(-t) = -3t - 4$

for $-1 \le t \le 3$

7. $x(t) = \sqrt{4-t}$, $y(t) = t + 1$ for $2 \le t \le 4$.

$x(-t) = \sqrt{4+t}$

$y(-t) = 1 - t$

for $-4 \le t \le -2$

8. $x(t) = |t + 5| + 1,\ y(t) = 3 - t$ for $-5 \le t \le -1$.

$x(-t) = |5 - t| + 1$

$y(-t) = 3 + t$

for $1 \le t \le 5$

9. $x(t) = \frac{t-1}{2},\ y(t) = \frac{2}{3}(t-2)(t+4)(1-t)$ for $-8 \le t \le 2$.

$x(-t) = \frac{-t-1}{2}$

$y(-t) = \frac{2}{3}(-t-2)(4-t)(1+t)$

for $-2 \le t \le 8$

10. $x(t) = t + 1,\ y(t) = t^2 - 4t + 1$ for $-10 \le t \le -3$.

$x(-t) = -t + 1$

$y(-t) = t^2 + 4t + 1$

for $3 \le t \le 10$

11. A particles motion in the plane is modeled by the parametric $x(t) = -\frac{1}{2}(t-2)^2$ and $y(t) = 2t + 3$.

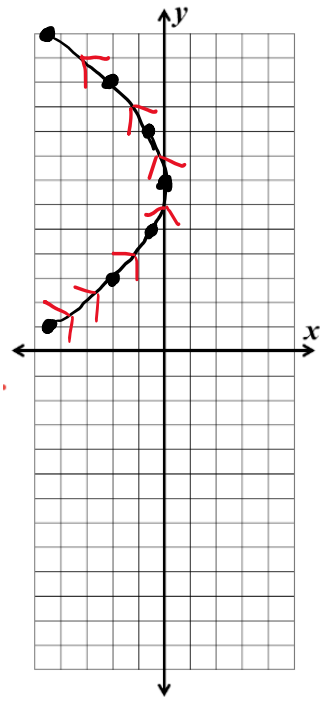
a. Fill in the table of values and use it to describe the direction of the particle motion over the interval $-1 \le t \le 5$. Assume the direction does not change between the values t shown in the table.

t	-1	0	1	2	3	4	5
x	$-\frac{9}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-\frac{9}{2}$
y	1	3	5	7	9	11	13

On the interval, $-1 \le t \le 2$, the direction is to the right and upward.

On the interval , $2 \le t \le 5$, the direction is to the left and upward.

b. Sketch a graph and indicate the direction of movement along the path.



Find the slope of the graph between the points that correspond to the given values of t.

12. $f(t) = (t + 1, 2t + 1)$ at $t = 1$ and $t = 5$

$y(5) = 11$ $\quad x(5) = 6$

$y(1) = 3$ $\quad x(1) = 2$

$$\frac{\frac{y(5)-y(1)}{5-1}}{\frac{x(5)-x(1)}{5-1}} = \frac{\frac{11-3}{4}}{\frac{6-2}{4}} = \frac{2}{1}$$

$\boxed{2}$

13. $f(t) = (-2|t + 4| - 3, t)$ at $t = -2$ and $t = 4$

$y(4) = 4$ $\quad x(4) = -2(8) - 3 = -19$

$y(-2) = -2$ $\quad x(-2) = -2(2) - 3 = -7$

$$\frac{\frac{y(4)-y(-2)}{4--2}}{\frac{x(4)-x(-2)}{4--2}} = \frac{\frac{4--2}{6}}{\frac{-19--7}{6}} = \frac{1}{-12/6}$$

$\boxed{-\frac{1}{2}}$