AP Precalc

Write your questions and thoughts here!

4.4 Parametrically Defined Circles and Lines

4.4 Notes

Today, we will look at how to parametrically express motion for particles that move around a circle or along a line segment.

Recall from Algebra 1: To find the equation of a line, we need to find the slope and use a point. When the point is the initial point (y-intercept), we can use slope-intercept form: $y =$ ____________.

Parametrically Defined Lines

First, there are MANY ways you can parameterize the linear path of a particle that is moving from one point to another point. One way to parameterize the linear path of a particle is to use…

- The initial position (x_1, y_1).
- The rates of changes for both $x(t)$ and $y(t)$.

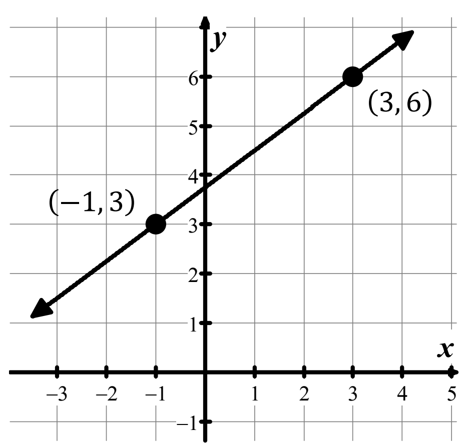
How do we find the initial position and rates of change? If we are given two points, do the following:

1. Our starting point will be when $t = 0$, which we designate as (x_1, y_1).
2. The second point will be when $t = 1$, which we designate as (x_2, y_2).
3. Find the average rate of change for both x and y.
$$\frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{1-0} \text{ and } \frac{\Delta y}{\Delta t} = \frac{y_2 - y_1}{1-0}$$
4. Our parametric equations can be written as follows: $\boldsymbol{x(t)} =$ ________ and $\boldsymbol{y(t)} =$ ________

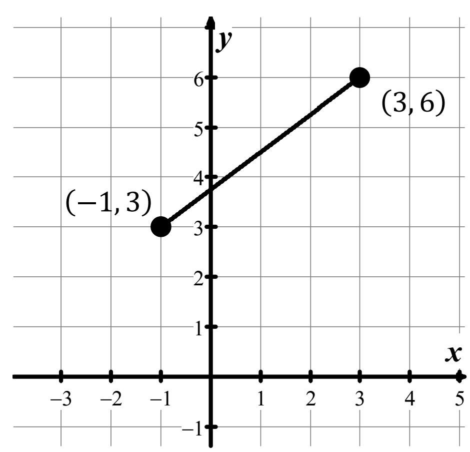
Ex 1: Find a set of parametric equations for the line that passes through the points $(-1,3)$ and $(3,6)$.

Verify our answer is correct by looking at the graph. Using Algebra 1 techniques, we find the slope $m = \frac{6-3}{3+1} = \frac{3}{4}$ and using our point $(-1, 3)$ and point-slope form we get $y - 3 = \frac{3}{4}(x + 1)$ or $y = \frac{3}{4}x + \frac{15}{4}$.

Algebra 1 Technique
$\boldsymbol{y = \frac{3}{4}x + \frac{15}{4}}$



Parameterization Technique
$\boldsymbol{x(t) = -1 + 4t}$ **and** $\boldsymbol{y(t) = 3 + 3t}$



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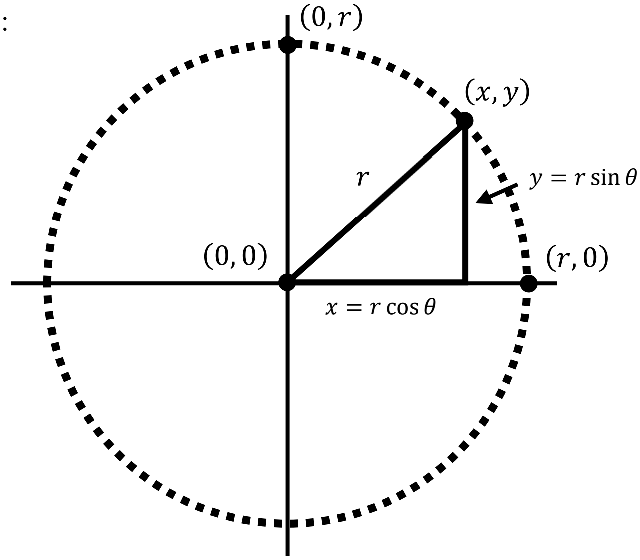
Parametrically Defined Circles

Recall from Geometry, Algebra 2, and Trigonometry (Unit 3 from this course).

- Equation of a circle centered at the origin is:

- The equation of a circle with the center at (h, k) is

- $x = r\cos\theta$
- $y = r\sin\theta$



Remember, parametric equations are used to describe the direction of particle motion, where $x(t)$ is the horizontal movement and $y(t)$ is the vertical movement. Using this knowledge, we can parameterize the equation of a circle as

Circle of radius $r = 1$, centered at the origin: $f(t) =$______________.

Note: This will always yield counterclockwise direction, swapping $\cos t$ and $\sin t$ would result in clockwise rotation, but that is not in this section.

Parametric Equation of a Circle

The parametric equations of a circle centered at (h, k) can be written as

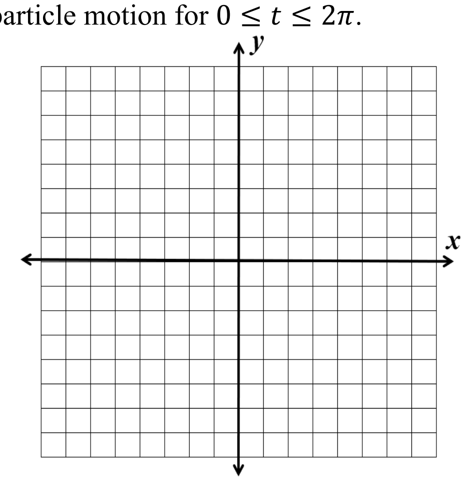
$\boldsymbol{x(t) =}$ **and** $\boldsymbol{y(t) =}$

where r is the radius of the circle.

Ex 2: Use the equation of the circle $(x-2)^2 + (y+4)^2 = 4$ to answer the questions below.

a. Find the parametric equations for the circle.

b. Graph the circle and indicate the direction of particle motion for $0 \le t \le 2\pi$.



4.4 Parametrically Defined Circles and Lines

AP Precalculus

4.4 Practice

1. Find the parametric equations for the linear path of a particle that travels from the point $(0,3)$ to the point $(-1,5)$.

2. If a particle traveling on a linear path has a starting point of $(3,6)$ and a slope of $\frac{1}{3}$, find the parametric equations that represent the linear path.

3. If the parametric equations for the linear path a particle travels are given by $(-1+7t, 5+4t)$, what is the slope of the path of the particle?

4. If the average rate of change of x is -2 and of y is 1, what are the parametric equations of a particle traveling on a linear path that starts at the origin?

5. Which of the following give the parametric equations for a particle traveling on a linear path that passes through the point $(1,3)$ and then the point $(-2,8)$?

   i. $x(t) = 1 - 3t$ and $y(t) = 3 + 5t$
   ii. $x(t) = 1 + 3t$ and $y(t) = 3 - 5t$
   iii. $x(t) = -2 - 3t$ and $y(t) = 8 + 5t$
   iv. $x(t) = -2 + 3t$ and $y(t) = 8 - 5t$

   (A) i only
   (B) i and iii
   (C) iii only
   (D) i and iv

6. Find the parametric equations for the circle with the center at $(-2, -1)$ and a radius of 6.

7. If a particle is traveling on a circular path and its distance from the origin at any moment of time is $\sqrt{2}$ units, find the parametric equations for this situation.

8. Find the parametric equations for a particle traveling on the path modeled by $(x + 2)^2 + (y - 6)^2 = 5$.

9. If a unit circle centered at the origin has a transformation of 3 units to the left and 5 units down, find the parameterization of the circle in the new location.

10. If the parametric equations $x(t) = 1 + 5\cos t$ and $y(t) = 2 + 5\sin t$ are used to describe the path a particle is traveling, find the rectangular form equation of the graph it creates.