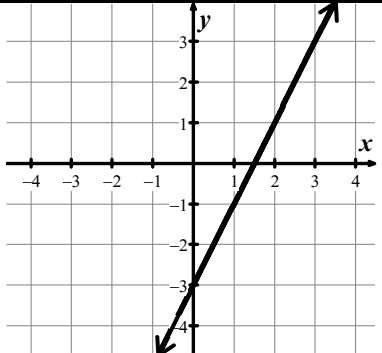
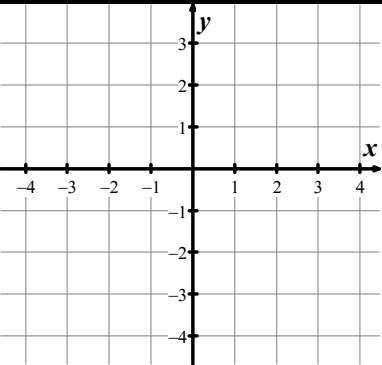


Write your questions
and thoughts here!

Explicit equations are what you have used through pre-algebra and algebra. Today, we will look at implicit equations as we prepare for conic sections. Compare the following equations in the table below.

Implicit	Explicit	Graph
$2x - y + 3 = 0$	$y = 2x - 3$	
$x^2 + y^2 - 4 = 0$	and	

Explicit functions can be written $y = \underline{\hspace{2cm}}$ and implicit functions as $f(x, y) = \underline{\hspace{2cm}}$.

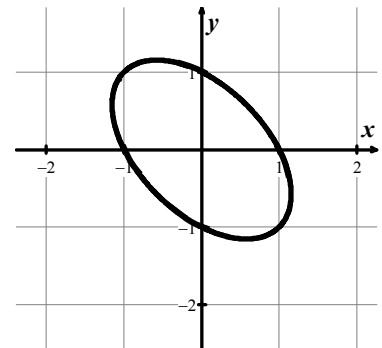
Given the following implicit equations, solve for y to create one or more functions that produce the same graph.

1. $x^2 + y^2 - 16 = 0$

2. $x^2 - 2xy + y + 6 = 0$

It is not always easy or even possible to solve an implicitly defined function for one variable. For example, $x^2 + xy + y^2 - 1 = 0$.

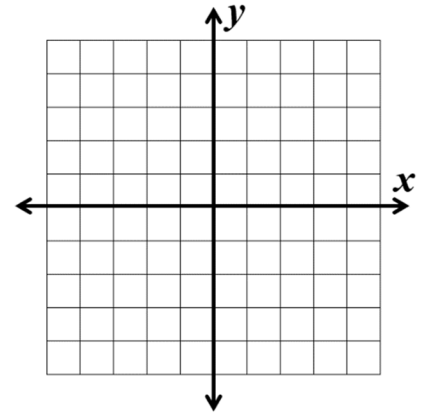
How could we graph implicitly defined functions? We can do it by finding solutions. Pick a value for one variable and solve for the other variable. The example $x^2 + xy + y^2 - 1 = 0$ would be fairly challenging, so let's do a simpler equation.



Write your questions and thoughts here!

3. Graph $4x^2 + 9y^2 = 36$ by finding several coordinate points to help you make a sketch on the given set of axes.

x	-3	0	3
y			



Varying Quantities for Implicit Equations

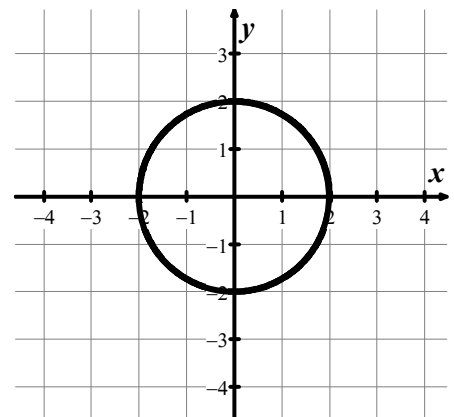
For ordered pairs on the graph that are close together of an implicitly defined function, if the ratio of the change in the two variables is positive, then the two variables simultaneously increase or both decrease.

$$\frac{\Delta y}{\Delta x} = \frac{\text{positive}}{\text{positive}} = \frac{\text{negative}}{\text{negative}} = \text{positive, then both variables simultaneously } \underline{\hspace{2cm}}.$$

If the ratio is negative, then one variable increases, and the other decreases.

$$\frac{\Delta y}{\Delta x} = \frac{\text{positive}}{\text{negative}} = \frac{\text{negative}}{\text{positive}} = \text{negative, then one variable } \underline{\hspace{2cm}}.$$

4. Determine how the variables x and y vary together in the **first quadrant** of the implicitly defined function $x^2 + y^2 = 4$, as the x -value goes from $x = 0$ to $x = 2$.



Vertical or Horizontal intervals

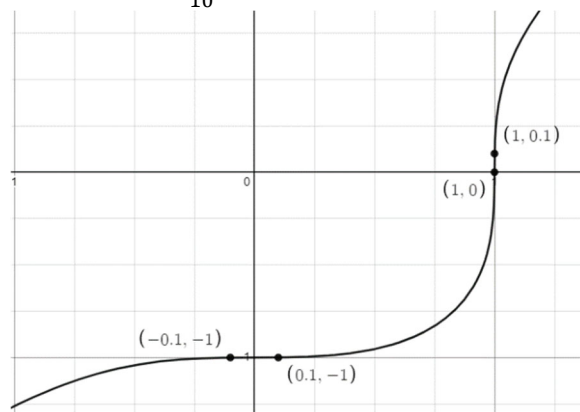
The rate of change of x with respect to y , or the rate of change of y with respect to x , can be 0. This indicates vertical or horizontal intervals, respectively. In other words,

$$\frac{\Delta y}{\Delta x} = 0 \text{ indicates a } \underline{\hspace{2cm}} \text{ interval}$$

$$\frac{\Delta x}{\Delta y} = 0 \text{ indicates a } \underline{\hspace{2cm}} \text{ interval}$$

This is simply an Algebra 1 idea! Think of the slope of horizontal and vertical lines.

5. Given the implicitly defined function $x^3 - y^3 - \frac{1}{100}xy - 1 = 0$. The following coordinate points are on the graph and are found in the figure below: $(-\frac{1}{10}, -1)$, $(\frac{1}{10}, -1)$, $(1, 0)$, and $(1, \frac{1}{10})$.
- a. Find the change of x with respect to y , on the interval $0 \leq y \leq \frac{1}{10}$.



- b. Find the change of y with respect to x , on the interval $-\frac{1}{10} \leq x \leq \frac{1}{10}$.

4.5 Implicitly Defined Functions

AP Precalculus

4.5 Practice

1. Which of the following sets of ordered pairs satisfy the given implicitly defined function $x^2 + y^2 - 4y - 5 = 0$?

- (A) $(0,5), (0, -1), (0, 3.236), (0, -3.236)$
- (B) $(0, -5), (0,1), (3.236, 0), (-3.236, 0)$
- (C) $(0, -5), (0,1), (2.236, 0), (-2.236, 0)$
- (D) $(0,5), (0, -1), (2.236, 0), (-2.236, 0)$

2. Match the implicitly defined function to the equivalent function, $(x - 2)^2 + (y + 1)^2 - 9 = 0$.

(A) $x = -2 \pm \sqrt{9 - (y + 1)^2}$

(B) $x = 2 \pm \sqrt{3 - (y + 1)}$

(C) $x = -2 \pm \sqrt{3 - (y + 1)^2}$

(D) $x = 2 \pm \sqrt{9 - (y + 1)^2}$

3. Rewrite $x = (8 - (y + 1)^2)^{\frac{1}{3}}$ as an implicitly defined function.

4. Identify the function whose graph is the “lower half” of the equation $y^2 + 3x - 1 = 0$.

5. Find the rate of change of y with respect to x and determine how the two quantities in the implicitly defined function, $x^2 + 9y^2 - 1 = 0$, vary together on the interval $0 \leq x \leq 1$, and $y \geq 0$.

6. Which of the following sets of ordered pairs found on the graph of an implicitly defined function would indicate a vertical interval in the graph? The points are considered close together.

(A) $(-2, 1), (4, 1)$

(B) $(1, 1.34), (1, 2.45)$

(C) $(2.2, 4.76)$

(D) $(5, 7), (-5, 7)$

(E) None of the above

7. The equation $x^2 + \frac{1}{2}y^2 - 4 = 0$ represents an ellipse.

a. Create an explicit equation that represents the top half of the ellipse.

b. Create an explicit equation that represents the left side of the ellipse.

8. Which of the following could be the ratio of change in the two variables of an implicitly defined function over an interval, if the two variables simultaneously decrease in the interval.

(A) $\frac{\Delta y}{\Delta x} = -\frac{2}{3}$

(B) $\frac{\Delta y}{\Delta x} = \frac{2}{3}$

(C) $\frac{\Delta y}{\Delta x} = 0$

(D) $\frac{\Delta x}{\Delta y} = 0$

(E) None of the above

9. Complete the table of numerical values and graph the implicitly defined function $(x - 1)^2 + y^2 - 9 = 0$. The graph will have the shape of an **ellipse**. Clearly label all your points.

x	-2	1	4
y			

