

4.5 Implicitly Defined Functions

AP Precalculus

Solutions

4.5 Practice

1. Which of the following sets of ordered pairs satisfy the given implicitly defined function $x^2 + y^2 - 4y - 5 = 0$?

$$(0, 5) \rightarrow 0 + 25 - 20 - 5 = 0$$

$$0 = 0 \quad \checkmark$$

(A) $(0, 5), (0, -1), (0, 3.236), (0, -3.236)$

D (B) $(0, -5), (0, 1), (3.236, 0), (-3.236, 0)$ $(0, -5) \rightarrow 0 + 25 + 20 - 5 = 0$

$$40 = 0 \quad \times$$

(C) $(0, -5), (0, 1), (2.236, 0), (-2.236, 0)$

(D) $(0, 5), (0, -1), (2.236, 0), (-2.236, 0)$

$$(2.236, 0) \rightarrow (2.236)^2 + 0 - 0 - 5 = 0$$

$$-0.0003 \approx 0 \quad \checkmark$$

2. Match the implicitly defined function to the equivalent function, $(x - 2)^2 + (y + 1)^2 - 9 = 0$.

Solve for x

(A) $x = -2 \pm \sqrt{9 - (y + 1)^2}$

(B) $x = 2 \pm \sqrt{3 - (y + 1)^2}$

(C) $x = -2 \pm \sqrt{3 - (y + 1)^2}$

(D) $x = 2 \pm \sqrt{9 - (y + 1)^2}$

$$(x-2)^2 = 9 - (y+1)^2$$

$$x-2 = \pm \sqrt{9 - (y+1)^2}$$

D

3. Rewrite $x = (8 - (y + 1)^2)^{\frac{1}{3}}$ as an **implicitly** defined function.

$$x^3 = 8 - (y+1)^2$$

$$x^3 + (y+1)^2 - 8 = 0$$

4. Identify the function whose graph is the “lower half” of the equation $y^2 + 3x - 1 = 0$.

$$y^2 = -3x + 1$$

$$y = \pm \sqrt{-3x + 1}$$

$$y = -\sqrt{-3x + 1}$$

5. Find the rate of change of y with respect to x and determine how the two quantities in the implicitly defined function, $x^2 + 9y^2 - 1 = 0$, vary together on the interval $0 \leq x \leq 1$, and $y \geq 0$.

Find y when $x=0$

$$0 + 9y^2 = 1$$

$$y^2 = \frac{1}{9}$$

$$y = \pm \frac{1}{3}$$

use $y = \frac{1}{3}$ because $y \geq 0$.

$$(0, \frac{1}{3})$$

Find y when $x=1$

$$1^2 + 9y^2 - 1 = 0$$

$$9y^2 = 0$$

$$y = 0$$

$$(1, 0)$$

$$\frac{\Delta y}{\Delta x} = \frac{0 - \frac{1}{3}}{1 - 0} = \frac{-\frac{1}{3}}{1} = -\frac{1}{3}$$

As one variable increases, the other decreases.

6. Which of the following sets of ordered pairs found on the graph of an implicitly defined function would indicate a vertical interval in the graph? The points are considered close together.

(A) $(-2, 1), (4, 1)$

(B) $(1, 1.34), (1, 2.45)$

(C) $(2.2, 4.76)$

(D) $(5, 7), (-5, 7)$

(E) None of the above

B

A vertical interval means $\frac{\Delta y}{\Delta x}$ is undefined, or $\frac{\Delta x}{\Delta y} = 0$.

This means the two x -values should be the same in the two coordinate points.

7. The equation $x^2 + \frac{1}{2}y^2 - 4 = 0$ represents an ellipse.

a. Create an explicit equation that represents the top half of the ellipse.

$$\begin{aligned} \frac{1}{2}y^2 &= 4 - x^2 \\ y^2 &= 8 - 2x^2 \\ y &= \pm \sqrt{8 - 2x^2} \end{aligned}$$

$$y = \sqrt{8 - 2x^2}$$

$y =$

b. Create an explicit equation that represents the left side of the ellipse.

$$\begin{aligned} x^2 &= 4 - \frac{1}{2}y^2 \\ x &= \pm \sqrt{4 - \frac{1}{2}y^2} \end{aligned}$$

$$x = -\sqrt{4 - \frac{1}{2}y^2}$$

$x = -$

8. Which of the following could be the ratio of change in the two variables of an implicitly defined function over an interval, if the two variables simultaneously decrease in the interval.

$\frac{\text{neg}}{\text{neg}} = \text{positive rate of change}$

(A) $\frac{\Delta y}{\Delta x} = -\frac{2}{3}$

(B) $\frac{\Delta y}{\Delta x} = \frac{2}{3}$

(C) $\frac{\Delta y}{\Delta x} = 0$

(D) $\frac{\Delta x}{\Delta y} = 0$

(E) None of the above

9. Complete the table of numerical values and graph the implicitly defined function $(x - 1)^2 + y^2 - 9 = 0$. The graph will have the shape of a **circle**. Clearly label all your points.

x	-2	1	4
y	0	-3 and 3	0

$(1, -3)$ $(1, 3)$

$$\begin{aligned} x = -2 \rightarrow (-2-1)^2 + y^2 - 9 &= 0 \\ 9 + y^2 - 9 &= 0 \\ y &= 0 \end{aligned}$$

$$\begin{aligned} x = 1 \rightarrow (1-1)^2 + y^2 - 9 &= 0 \\ y^2 &= 9 \\ y &= \pm 3 \end{aligned}$$

$$\begin{aligned} x = 4 \rightarrow (4-1)^2 + y^2 - 9 &= 0 \\ 9 + y^2 - 9 &= 0 \\ y &= 0 \end{aligned}$$

