

4.6B Conic Sections: Ellipses

AP Precalculus

4.6B Practice

1. Use the equation $\frac{(x+2)^2}{36} + \frac{(y-5)^2}{4} = 1$ to find the following.

a. center

$$(-2, 5)$$

b. horizontal or vertical

horizontal

c. vertices (ends of major axis)

$$\begin{aligned} a^2 &= 36 \\ a &= 6 \\ x &= -2 \pm 6 \\ y &= 5 \end{aligned}$$

$$(-8, 5) \text{ and } (4, 5)$$

d. ends of minor axis

$$\begin{aligned} b^2 &= 4 \\ b &= 2 \\ x &= -2 \\ y &= 5 \pm 2 \end{aligned}$$

$$(-2, 3) \text{ and } (-2, 7)$$

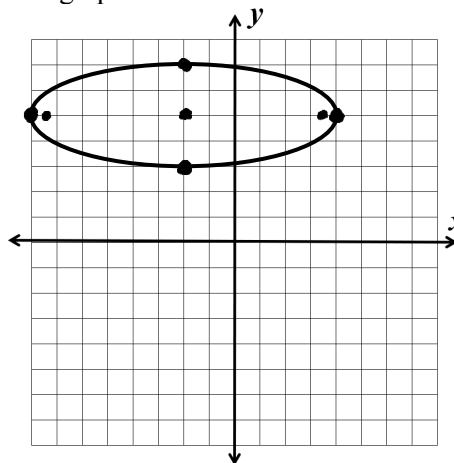
e. foci

$$\begin{aligned} c^2 &= 36 - 4 \\ c^2 &= 32 \\ c &= \sqrt{32} \approx 5.657 \end{aligned}$$

$$\begin{aligned} x &= -2 \pm \sqrt{32} \\ y &= 5 \end{aligned}$$

$$\begin{aligned} &(-2 \pm \sqrt{32}, 5) \\ &\text{or} \\ &(-7.657, 5) \text{ and } (3.657, 5) \end{aligned}$$

f. sketch the graph



2. Use the equation $\frac{(x+5)^2}{16} + \frac{y^2}{49} = 1$ to find the following.

a. center

$$(-5, 0)$$

b. horizontal or vertical

vertical

c. vertices (ends of major axis)

$$\begin{aligned} b^2 &= 49 \\ b &= 7 \\ x &= -5 \\ y &= 0 \pm 7 \end{aligned}$$

$$(-5, -7) \text{ and } (-5, 7)$$

d. ends of minor axis

$$\begin{aligned} a^2 &= 16 \\ a &= 4 \\ x &= -5 \pm 4 \\ y &= 0 \end{aligned}$$

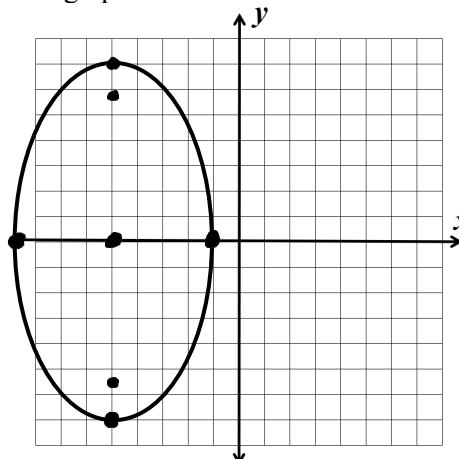
$$(-9, 0) \text{ and } (-1, 0)$$

e. foci

$$\begin{aligned} c^2 &= 49 - 16 \\ c &= \sqrt{33} \approx 5.744 \\ x &= -5 \\ y &= 0 \pm \sqrt{33} \end{aligned}$$

$$\begin{aligned} &(-5, \pm\sqrt{33}) \\ &\text{or} \\ &(-5, -5.744) \text{ and } (-5, 5.744) \end{aligned}$$

f. sketch the graph



3. Find the equation of an ellipse, in standard form, with a vertical orientation, center at $(0,4)$, major axis length of 22 and a minor axis length of 14.

$$\begin{aligned} 2b &= 22 & 2a &= 14 \\ b &= 11 & a &= 7 \\ b^2 &= 121 & a^2 &= 49 \end{aligned}$$

$$\frac{x^2}{49} + \frac{(y-4)^2}{121} = 1$$

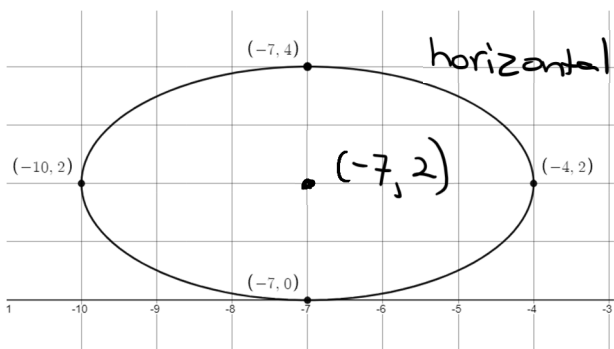
4. Find the equation of an ellipse, in standard form, with a vertical orientation, center at $(-1, -5)$, Foci at $(-1, -3)$, $(-1, -7)$ and a minor axis length of 6.

$$\begin{aligned} c &= |-7 - (-5)| & 2a &= 6 \\ c &= 2 & a &= 3 \\ c^2 &= 4 & a^2 &= 9 \\ c^2 &= b^2 - a^2 \\ 4 &= b^2 - 9 \\ 13 &= b^2 \end{aligned}$$

$$\frac{(x+1)^2}{9} + \frac{(y+5)^2}{13} = 1$$

Match the graph with its equation.

5.



(A) $\frac{(x+7)^2}{9} + \frac{(y-2)^2}{4} = 1$

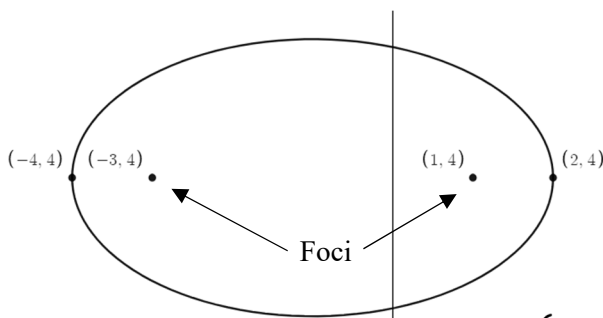
(B) $\frac{(x+7)^2}{4} + \frac{(y-2)^2}{9} = 1$

(C) $\frac{(x-7)^2}{4} + \frac{(y+2)^2}{9} = 1$

(D) $\frac{(x-7)^2}{9} + \frac{(y+2)^2}{4} = 1$

$$\begin{aligned} a &= 3 \\ a^2 &= 9 \\ b &= 2 \\ b^2 &= 4 \end{aligned}$$

6.



(A) $\frac{(x+1)^2}{9} + \frac{(y-4)^2}{5} = 1$

(B) $\frac{(x+1)^2}{5} + \frac{(y-4)^2}{9} = 1$

(C) $\frac{(x-1)^2}{5} + \frac{(y+4)^2}{9} = 1$

(D) $\frac{(x-1)^2}{5} + \frac{(y+4)^2}{4} = 1$

$$\begin{aligned} \text{Center: } &(-1, 4) \\ c &= |-1 - 1| = 2 \\ c^2 &= 4 \\ a &= 2 - (-1) = 3 \\ a^2 &= 9 \\ c^2 &= a^2 - b^2 \\ 4 &= 9 - b^2 \\ b^2 &= 5 \end{aligned}$$

Put the given equation of an ellipse into standard form. Then identify the center, foci, and orientation.

7. $x^2 + 3y^2 + 6x - 12y + 6 = 0$

$$x^2 + 6x + 3y^2 - 12y = -6$$

$$x^2 + 6x + 9 + 3(y^2 - 4y + 4) = -6 + 9 + 12$$

$$(x+3)^2 + 3(y-2)^2 = 15$$

$$\frac{(x+3)^2}{15} + \frac{(y-2)^2}{5} = 1$$

8. $3x^2 + y^2 - 18x - 6 = 0$

$$3x^2 - 18x + y^2 = 6$$

$$3(x^2 - 6x + 9) + y^2 = 6 + 27$$

$$3(x-3)^2 + y^2 = 33$$

$$\frac{(x-3)^2}{11} + \frac{y^2}{33} = 1$$

9. Using the equation $4(x + 9)^2 + (y + 2)^2 = 16$, find the center, foci and vertices of the ellipse.

$$\frac{(x+9)^2}{4} + \frac{(y+2)^2}{16} = 1$$

↑ vertical $b^2 = 16$
 $b = 4$

$$c^2 = 16 - 4$$

$$c = \sqrt{12} \approx 3.464$$

Center: $(-9, -2)$

Vertices: $(-9, -6)$ and $(-9, 2)$

Foci: $(-9, -2 - \sqrt{12})$ and $(-9, -2 + \sqrt{12})$

10. Given the foci of an ellipse are located at $(2, 5)$ and $(2, 7)$ and the minor axis length is 10, find the equation of the ellipse in standard form.

center: $(2, 6)$ $2a = 10$
 vertical $a = 5$
 $c = 1$ $a^2 = 25$
 $c^2 = 1$

$$c^2 = b^2 - a^2$$

$$1 = b^2 - 25$$

$$26 = b^2$$

$$\frac{(x - 2)^2}{25} + \frac{(y - 6)^2}{26} = 1$$