4.6C Conic Sections: Hyperbolas

AP Precalculus

Solutions

4.6C Practice

1. Use the equation $\frac{(x+1)^2}{25} - \frac{(y-3)^2}{16} = 1$ to find the following.

a. center

$(-1, 3)$

b. horizontal/vertical

horizontal

c. Find the length of the transverse axis.

$a^2 = 25$

$a = 5$

10

d. Find the length of the conjugate axis.

$b^2 = 16$

$b = 4$

8

e. vertices

$(-1, 3)$

± 5

$(-6, 3)$ and $(4, 3)$

f. foci

$c^2 = 25 + 16$

$c^2 = 41$

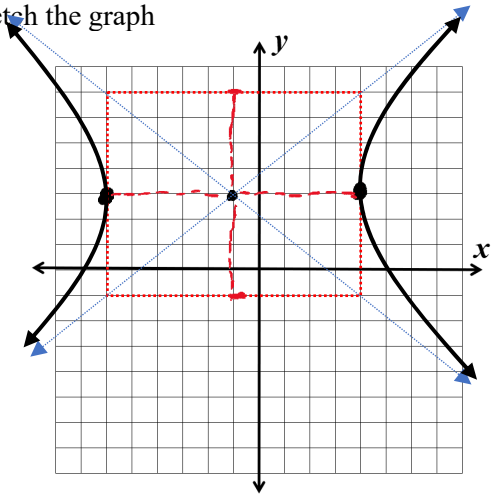
$c = \sqrt{41}$

$(-1 - \sqrt{41}, 3)$ and $(-1 + \sqrt{41}, 3)$

or

$(-7.403, 3)$ and $(5.403, 3)$

g. sketch the graph



2. Use the equation $\frac{(y+1)^2}{16} - \frac{x^2}{49} = 1$ to find the following.

a. center

$(0, -1)$

b. horizontal/vertical

vertical

c. Find the length of the transverse axis.

$b^2 = 16$

$b = 4$

8

d. Find the length of the conjugate axis.

$a^2 = 49$

$a = 7$

14

e. vertices

$(0, -1)$

± 4

$(0, -5)$ and $(0, 3)$

f. foci

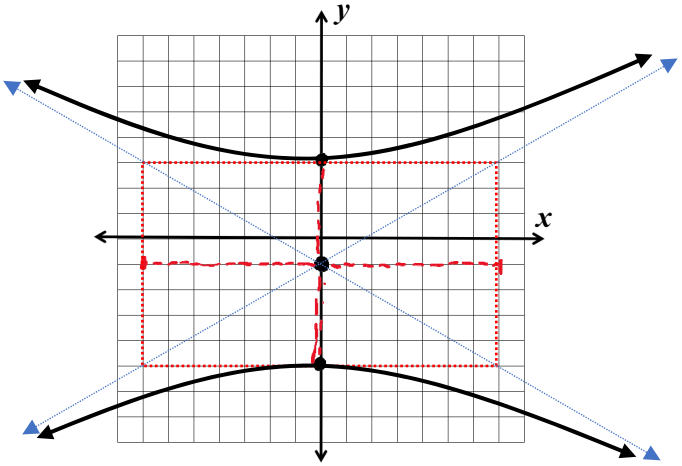
$c^2 = 16 + 49$

$c = \sqrt{64}$

$c = 8$

$(0, -9)$ and $(0, 7)$

g. sketch the graph



3. Find the equation of a hyperbola, in standard form, with a vertical orientation, center at $(0,4)$, transverse axis length of 22 and conjugate axis length of 14.

$2b = 22$ $\quad 2a = 14$
$b = 11$ $\quad a = 7$
$b^2 = 121$ $\quad a^2 = 49$

$$-\frac{x^2}{49} + \frac{(y-4)^2}{121} = 1$$

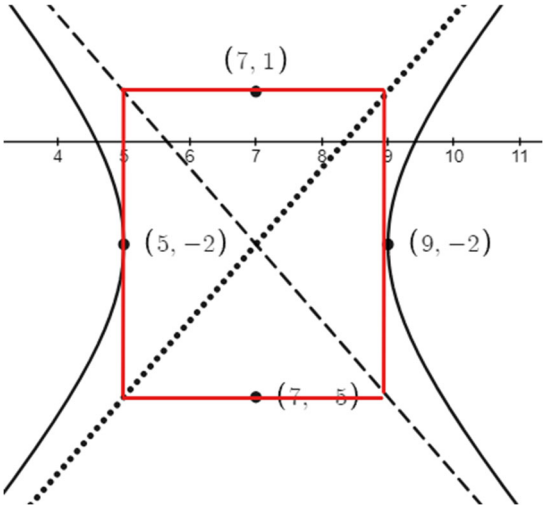
4. Find the equation of a hyperbola, in standard form, with a horizontal orientation, center at $(-2, 3)$, transverse axis length of 4 and conjugate axis length of 16.

$2a = 4$ $\quad 2b = 16$
$a = 2$ $\quad b = 8$
$a^2 = 4$ $\quad b^2 = 64$

$$\frac{(x+2)^2}{4} - \frac{(y-3)^2}{64} = 1$$

Match the graph with its equation.

5.



Center: $(7, -2)$ horizontal $\quad 2a = 4$, $a = 2$, $a^2 = 4$ $\quad 2b = 6$, $b = 3$, $b^2 = 9$

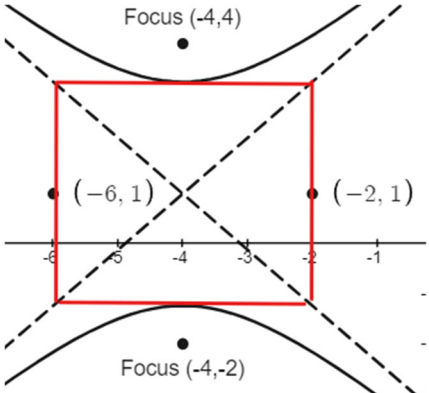
(A) $\frac{(x+7)^2}{9} - \frac{(y-2)^2}{4} = 1$

(B) $\frac{(x+7)^2}{4} - \frac{(y-2)^2}{9} = 1$

(C) $\frac{(x-7)^2}{4} - \frac{(y+2)^2}{9} = 1$ C

(D) $\frac{(x-7)^2}{9} - \frac{(y+2)^2}{4} = 1$

6.



Center: $(-4, 1)$ vertical $\quad 2c = 6$, $c = 3$, $c^2 = a^2 + b^2$, $9 = 4 + b^2$, $b^2 = 5$ $\quad 2a = 4$, $a = 2$, $a^2 = 4$

(A) $\frac{(y+1)^2}{4} - \frac{(x-4)^2}{5} = 1$

(B) $\frac{(y+1)^2}{5} - \frac{(x-4)^2}{4} = 1$

(C) $\frac{(y-1)^2}{4} - \frac{(x+4)^2}{5} = 1$

(D) $\frac{(y-1)^2}{5} - \frac{(x+4)^2}{4} = 1$ D

Put the given equation of a hyperbola into standard form. Then identify the center, foci, and vertices.

7. $x^2 - 3y^2 + 6x - 12y + 6 = 0$

$x^2+6x+9-3(y^2+4y+4) = -6+9-12$

$(x+3)^2 - 3(y+2)^2 = -9$

$$-\frac{(x+3)^2}{9} + \frac{(y+2)^2}{3} = 1$$

8. $3x^2 - y^2 - 18x - 6 = 0$

$3(x^2-6x+9) - (y^2) = 6+27$

$3(x-3)^2 - y^2 = 33$

$$\frac{(x-3)^2}{11} - -\frac{y^2}{33} = 1$$

9. Use the equation $3(x+9)^2 - 2(y+2)^2 = -18$ to find the center, foci and vertices of the hyperbola.

$$-\frac{(x+9)^2}{6} + \frac{(y+2)^2}{9} = 1$$

Vertical

Center: $(-9,-2)$

$b^2 = 9$

$b = 3$ ± 3

Vertices: $(-9,-5)$ and $(-9,1)$

$c^2 = 6+9$

$c = \sqrt{15}$

Foci: $(-9, -2 \pm \sqrt{15})$

10. Given the foci of a hyperbola are located at $(2,5)$ and $(2,17)$ and the conjugate axis length is 10, find the equation of the hyperbola in standard form.

Center: $(2,11)$

horizontal

$c = 6$

$c^2 = 36$

$2b = 10$

$b = 5$

$b^2 = 25$

$$\frac{(x-2)^2}{a^2} - \frac{(y-11)^2}{b^2} = 1$$

$36 = a^2 + 25$

$11 = a^2$

$$\frac{(x-2)^2}{11} - \frac{(y-11)^2}{25} = 1$$