AP Precalc

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4.7 Parametrization of Implicitly Defined Functions

4.7 Notes

Recall from section 4.1. An implicitly defined function can be parametrized to the form $(x(t), y(t))$. The equations for $x(t)$ and $y(t)$ find the x- and y-values, respectively, of the coordinate point.

Simple Parametrization Technique

If f is a function of x, then $y = f(x)$ can be parametrized as $(\boldsymbol{x(t)}, \boldsymbol{y(t)}) =$

If f is **invertible**, its inverse can be parametrized as $(\boldsymbol{x(t)}, \boldsymbol{y(t)}) =$ for an appropriate interval of t.

1. Use the function $f(x) = x^3 - 2$ to answer each part below.

a. Find the parametrization of the function.

b. Find the parametrization of the inverse of the function.

c. Using your answer from part *a*, parametrize the function $f(x)$ by letting $x(t) = 2t$.

PARABOLA Parametrization

- If the equation can be solved for y, solve for y and replace x with t. → $(t, y(t))$
- If the equation can be solved for x, solve for x and replace y with t. → $(x(t), t)$

This allows us to parametrize a quadratic (parabola) without too much difficulty.

Parametrize the following equations.

2. $x^2 - 2y = 0$

3. $\frac{x}{4} + y^2 - 1 = 0$

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ELLIPSE Parametrization

Standard form: $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$

Parametric equations for an ellipse:

$x(t)=$ and $y(t)=$ for $0 \le t \le 2\pi$.

Parametrize the following equations.

4. $\frac{(x-2)^2}{16}+\frac{(y-5)^2}{9}=1$

5. $\frac{(x+3)^2}{13}+\frac{(y-1)^2}{25}=1$

HYPERBOLA (left/right) Parametrization

Standard form: $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Parametric equations for a left/right hyperbola:

$x(t)=$ and $y(t)=$ for $0 \le t \le 2\pi$.

HYPERBOLA (up/down) Parametrization

Standard form: $\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$

Parametric equations for an up/down hyperbola:

$x(t)=$ and $y(t)=$ for $0 \le t \le 2\pi$.

Parametrize the following equations.

6. $\frac{(x+1)^2}{144}-\frac{(y-4)^2}{25}=1$

7. $\frac{(y+3)^2}{36}-\frac{(x-1)^2}{4}=1$

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4.7 Parametrization of Implicitly Defined Functions

AP Precalculus

4.7 Practice

Instructions

1. Find the parameterization of the equation given by $y = 2x^2 - x + 1$, by letting $x(t) = t - 4$.

2. Which of the following could represent a parametrization of the equation $y = 2x + 1$?

   I. $(t, 2t + 1)$
   II. $(2t, 2t + 1)$
   III. $(2t, 4t + 2)$
   IV. $(2t, 4t + 1)$

   (A) I only (B) II only (C) III only

   (D) I and III only (E) I and IV only

3. Find the rectangular equation represented by the parametric equations $x(t) = t - 2$ and $y(t) = t^2 - 5$.

4. If the inverse of a function is $f^{-1}(x) = \sqrt[3]{x + 5}$, then which of the following represent parametric equations for the inverse?

   (A) $\left(\sqrt[3]{t + 5}, t\right)$ and $(t^3 - 5, t)$

   (B) $\left(t, \sqrt[3]{t + 5}\right)$ and $(t^3 - 5, t)$

   (C) $\left(t, \sqrt[3]{t + 5}\right)$ and $(t, t^3 - 5)$

   (D) $\left(\sqrt[3]{t - 5}, t\right)$ and $(t, t^3 + 5)$

5. Which of the following parametric equations represents the rectangular equation $y = \sqrt{x - 1}$, when we use the parameter $t = 3 + 2x$?

   (A) $x(t) = 3 + 2t, y(t) = \sqrt{t - 1}, t \geq 5$

   (B) $x(t) = t, y(t) = 3 + 2t, t \geq 5$

   (C) $x(t) = t, y(t) = \sqrt{t - 1}, t \geq 5$

   (D) $x(t) = \frac{1}{2}(t - 3), y(t) = \sqrt{\frac{1}{2}(t - 5)}, t \geq 5$

6. Find the rectangular equation represented by the parametric equations $x(t) = \cos t$ and $y(t) = 1 - \sin^2 t$, for $0 \leq t \leq 2\pi$.
   Hint: remember your trig identities.

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Identify the conic section and find a parametrization of the equation.

7. $y^2 + y + 1 - x = 0$

8. $3(x+1)^2 = 18 + (y+2)^2$

9. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

10. $\frac{(x-5)^2}{36} + \frac{(y+4)^2}{7} = 1$

11. $9x^2 + y^2 + 2y - 8 = 0$

12. $x^2 - 3y^2 + 6x - 12y + 6 = 0.$

Find the equation in rectangular form of each of the following parametric equations.

13. Ellipse: $(2 + 5\cos t\,, 4 + 6\sin t)$

14. Hyperbola: $\left(1 + 3\tan t\,, 4 + \sqrt{5}\sec t\right)$

15. Parabola: $(t, t^2 + 3t - 4)$

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