4.7 Parametrization of Implicitly Defined Functions

AP Precalculus

4.7 Practice

Instructions

1. Find the parameterization of the equation given by $y = 2x^2 - x + 1$, by letting $x(t) = t - 4$.

$(t, 2t^2 - t + 1)$

$\downarrow$

$(t-4, 2(t-4)^2 - (t-4) + 1)$

$(t-4, 2t^2 - 16t + 32 - t + 4 + 1)$

$\boxed{(t-4, 2t^2 - 17t + 37)}$

2. Which of the following could represent a parametrization of the equation $y = 2x + 1$?

I. $(t, 2t + 1)$
II. $(2t, 2t + 1)$
III. $(2t, 4t + 2)$
IV. $(2t, 4t + 1)$

$t, 2t+1$

$2t, 2(2t)+1$

$2t, 4t+1$

(A) I only (B) II only (C) III only

(D) I and III only (E) I and IV only

3. Find the rectangular equation represented by the parametric equations $x(t) = t - 2$ and $y(t) = t^2 - 5$.

Substitution!

$x = t - 2$

$x + 2 = t \rightarrow y = (x+2)^2 - 5$

$y = x^2 + 4x + 4 - 5$

$\boxed{y = x^2 + 4x - 1}$

4. If the inverse of a function is $f^{-1}(x) = \sqrt[3]{x + 5}$, then which of the following represent parametric equations for the inverse?

$x = \sqrt[3]{y+5}$

$x^3 - 5 = y$

(A) $(\sqrt[3]{t + 5}, t)$ and $(t^3 - 5, t)$

(B) $(t, \sqrt[3]{t + 5})$ and $(t^3 - 5, t)$

(C) $(t, \sqrt[3]{t + 5})$ and $(t, t^3 - 5)$

(D) $(\sqrt[3]{t - 5}, t)$ and $(t, t^3 + 5)$

5. Which of the following parametric equations represents the rectangular equation $y = \sqrt{x - 1}$, when we use the parameter $t = 3 + 2x$?

$t - 3 = 2x$

$\frac{1}{2}(t-3) = x$

$\sqrt{\frac{1}{2}(t-3) - 1}$

$\sqrt{\frac{1}{2}t - \frac{3}{2} - 1} = \sqrt{\frac{1}{2}t - \frac{5}{2}} = \sqrt{\frac{1}{2}(t-5)}$

(A) $x(t) = 3 + 2t, y(t) = \sqrt{t - 1}, t \geq 5$

(B) $x(t) = t, y(t) = 3 + 2t, t \geq 5$

(C) $x(t) = t, y(t) = \sqrt{t - 1}, t \geq 5$

(D) $x(t) = \frac{1}{2}(t - 3), y(t) = \sqrt{\frac{1}{2}(t - 5)}, t \geq 5$

6. Find the rectangular equation represented by the parametric equations $x(t) = \cos t$ and $y(t) = 1 - \sin^2 t$, for $0 \leq t \leq 2\pi$.
Hint: remember your trig identities.

$1 - \sin^2 t = \cos^2 t$

$x(t) = \cos t \quad y(t) = \cos^2 t$

Substitute

$\boxed{y = x^2}$

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Identify the conic section and find a parametrization of the equation.

7. $y^2 + y + 1 - x = 0$

$x = y^2 + y + 1$

Parabola

$(t^2 + t + 1, t)$

8. $3(x+1)^2 = 18 + (y+2)^2$

$3(x+1)^2 - (y+2)^2 = 18$

$\frac{(x+1)^2}{6} - \frac{(y+2)^2}{18} = 1$

hyperbola left/right

$(-1 + \sqrt{6}\sec t, -2 + \sqrt{18}\tan t)$

9. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

ellipse

$(3\cos t, 2\sin t)$

10. $\frac{(x-5)^2}{36} + \frac{(y+4)^2}{7} = 1$

ellipse

$(5 + 6\cos t, -4 + \sqrt{7}\sin t)$

11. $9x^2 + y^2 + 2y - 8 = 0$

$9x^2 + y^2 + 2y + 1 = 8 + 1$

$9x^2 + (y+1)^2 = 9$

$x^2 + \frac{(y+1)^2}{9} = 1$

ellipse

$(\cos t, -1 + 3\sin t)$

12. $x^2 - 3y^2 + 6x - 12y + 6 = 0.$

$x^2 + 6x + 9 - 3(y^2 + 4y + 4) = -6 + 9 - 12$

$(x+3)^2 - 3(y+2)^2 = -9$

$-\frac{(x+3)^2}{9} + \frac{(y+2)^2}{3} = 1$

hyperbola up/down

$(-3 + 3\tan t, -2 + \sqrt{3}\sec t)$

Find the equation in rectangular form of each of the following parametric equations.

13. Ellipse: $(2 + 5\cos t, 4 + 6\sin t)$

$\frac{(x-2)^2}{25} + \frac{(y-4)^2}{36} = 1$

14. Hyperbola: $(1 + 3\tan t, 4 + \sqrt{5}\sec t)$

y is positive

$\frac{(y-4)^2}{5} - \frac{(x-1)^2}{9} = 1$

15. Parabola: $(t, t^2 + 3t - 4)$

$y = x^2 + 3x - 4$

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