Name: Solutions Date:________________ Period: ______

1A Review

Unit 1A Review – Polynomial and Rational Functions

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets for lessons 1.1 – 1.6.

1. For the function $s(t)$, s is the number of people swimming at the local pool and t is the temperature measured in Fahrenheit. Identify the dependent and independent variables.

   Dep: The number of people swimming.

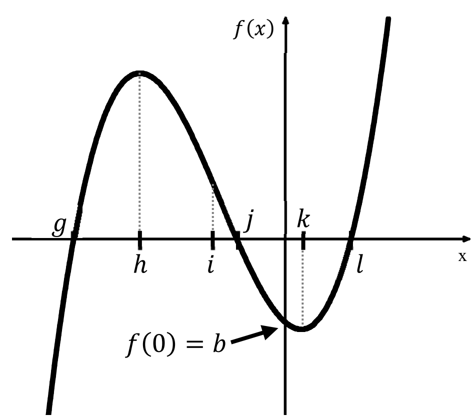
   Indep: The temperature (in Fahrenheit).

2. Let the function f be increasing or decreasing, but not both. State whether the function is increasing or decreasing on the interval $9 < x < 17$ and justify your answer.

x	9	11	13	15	17
$f(x)$	8	12	15	17	18

   f is increasing on the interval $9 < x < 17$ because for all a and b in the interval, if $a < b$, then $f(a) < f(b)$.

3. Use the graph to the right to answer the questions below.



   a. On what interval(s) is the graph concave up?

   $x > i$

   b. On what interval(s) is the graph concave down?

   $x < i$

   c. On what interval(s) is the graph increasing?

   $x < h$ and $x > k$

   d. On what interval(s) is the graph decreasing?

   $h < x < k$

   e. Find the zeros of the function.

   $x = g, j$ and l

   f. Find the y-intercept of the function.

   $y = b$

4. **Calculator active.** Find the average rate of change of the function $w(r) = \sqrt{1 - 4r}$ on the interval $-5 \le r \le -1$.

$$\frac{w(-1) - w(-5)}{-1 - -5} = \frac{\sqrt{5} - \sqrt{21}}{4} \approx -0.5866$$

5. **Calculator active.** Using the information in the table below, find the average rate of change for each given interval. Include units.

t seconds	24	60	135	200
$s(t)$ miles	8	1.3	3.5	0.3

a. $135 \le t \le 200$

$$\frac{0.3 - 3.5}{200 - 135}$$

-0.049 miles per second

b. $24 \le t \le 135$

$$\frac{3.5 - 8}{135 - 24}$$

-0.0405 miles per second

c. $24 \le t \le 200$

$$\frac{0.3 - 8}{200 - 24}$$

-0.0437 miles per second

6. **Calculator active.** Estimate the rate of change of $f(x) = x^2 - x$ at $x = -1$

$$\frac{f(-0.999) - f(-1)}{-0.999 - -1} = \boxed{-2.999}$$

7. Mr. Gardener is decreasing the amount of water used on his lawn each month, and the height of his grass is decreasing. Does this scenario represent a positive or negative rate of change?

Positive

What is the average rate of change for each function on the given intervals?

8. $y = 7 - 2x$ on $-4 \le x \le 1$

$y(1) = 5$

$y(-4) = 7 + 8 = 15$

$$\frac{5 - 15}{1 - -4} = \frac{-10}{5}$$

$\boxed{-2}$

9. $y = 3x^2 - 2x + 1$ on $-1 \le x \le 2$

$y(-1) = 3 + 2 + 1 = 6$

$y(2) = 12 - 4 + 1 = 9$

$$\frac{9 - 6}{2 - -1} = \frac{3}{3}$$

$\boxed{1}$

What is the rate of change of the average rates of change for each function over consecutive equal-length intervals?

10. $y = 5x + 9$

The average rate of change of the rates of change of a linear function is always **ZERO**.

11. $f(x) = 2x - 5x^2$.

$\mathbf{f(-1) = -2 - 5 = -7}$ > +7 > −10

$\mathbf{f(0) = 0}$ > −3

$\mathbf{f(1) = 2 - 5 = -3}$ > −13 > −10

$\mathbf{f(2) = 4 - 20 = -16}$

Rate of change is changing by -10.

$\boxed{\mathbf{-10}}$

12. The values of a function are given at selected x-values in the table below. The function's concavity does not change. Determine if the function is concave up or concave down. Justify your answer.

	4		4		4		4	
x	5		9		13		17		21
$g(x)$	45		20		0		−10		−14

−25 −20 −10 −4

Concave up because the rate of change is increasing over equal-length input-value intervals.

Find the leading coefficient and the degree of each polynomial.

13. $f(x) = x^5 - 2x^2$

L.C. 1 Degree: 5

14. $f(x) = 10 - 3x^2 + 7x^3 - 2x$

L.C. 7 Degree: 3

Let $f(x)$ be a polynomial function with the given values. Are there any guaranteed extrema? If so, state where they occur.

15. $f(0) = -3, f(4) = 0$, and $f(7) = 0$.

Yes, on the interval $4 < x < 7$.

16. $f(-7) = 0, f(-2) = 5, f(0) = 1$, and $f(9) = 0$.

Yes, on the interval $-7 < x < 9$.

Is there a global maximum or minimum for each function?

17. $f(x) = -5x^6 + 6x^4 - 3x^3 + 1$

Even degree, negative leading coefficient = opens down.

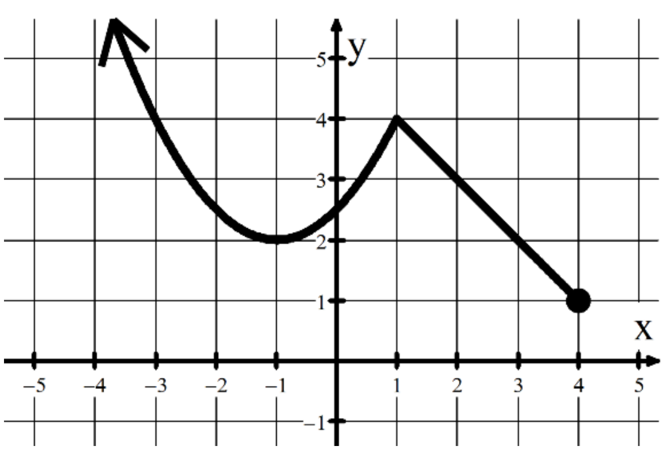
maximum

18. $f(x) = 2x^5 + x^2 - 6$

Odd degree

No absolute max or min

Find the following extrema. If there are none, cross it off and write NONE.



19. Absolute **min** of 1 when x = 4

20. ~~Absolute **max** of _____ when x =~~ none

21. Relative **min**(s) at x = −1 and x = 4

22. Relative **max**(es) at x = 1

For each polynomial function, find the intervals for each condition.

23. $h(x) = x^2 - 8x + 15$. When is $h(x) \leq 0$?

$0 = (x-3)(x-5)$

$x = 3 \qquad x = 5$

x	$(-\infty, 3)$	3	$(3, 5)$	5	$(5, \infty)$
$f(x)$	pos	0	neg	0	pos

$\boxed{[3, 5]}$

24. $f(x) = -x^3 + 5x^2 + 24x$. When is $f(x) \geq 0$?

$0 = -x(x^2 - 5x - 24)$

$0 = -x(x+3)(x-8)$

$x = 0 \qquad x = -3 \qquad x = 8$

x	$(-\infty, -3)$	-3	$(-3, 0)$	0	$(0, 8)$	8	$(8, \infty)$
$f(x)$	pos	0	neg	0	pos	0	neg

$\boxed{(-\infty, -3] \cup [0, 8]}$

25. The degree of a polynomial is 7 with real zeros at $x = -8, x = 1$, and $x = 4$. $x = 1$ has a multiplicity of 3. How many non-real zeros does the polynomial have?

Degree 7 − Real 5 =

$\boxed{\text{2 non-real Zeros}}$

26. $5 - i$ is a non-real zero of a polynomial, find another zero.

$\boxed{\mathbf{5 + i}}$

27. Find the **degree** of the polynomial from the given input and output values.

Input	0	1	2	3	4	5	6	7
Output	2	−2	4	20	46	82	128	184

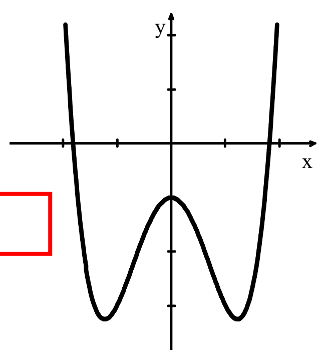
1st difference → −4 6 16 26 36 46 56

2nd difference → 10 10 10 10 10 10

$\boxed{\textbf{Degree} = \mathbf{2}}$

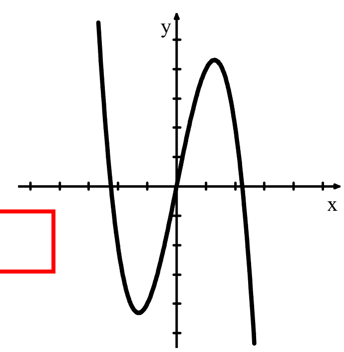
State whether the following graphs represent functions that are even, odd, or neither.

28.



EVEN

29.



ODD

State if the following functions are even, odd, or neither.

30. $f(x) = 4x^7 + 5x^3 - 2x$

$f(-x) = 4(-x)^7 + 5(-x)^3 - 2(-x)$

$f(-x) = -4x^7 - 5x^3 + 2x = -f(x)$

ODD

31. $f(x) = 7 - 6x^8 - 3x^2$

$f(-x) = 7 - 6(-x)^8 - 3(-x)^2$

$f(-x) = 7 - 6x^8 - 3x^2 = f(x)$

EVEN

Describe the end behavior of each function using limit notation.

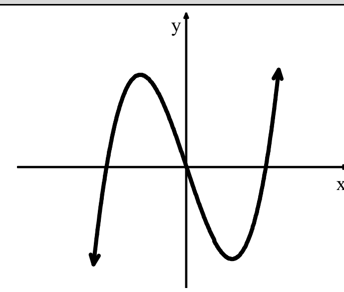
32. $p(x) = -11x^7 - 6x^2 + 4x$

$\lim_{x\to-\infty} p(x) = \infty$ **and** $\lim_{x\to\infty} p(x) = -\infty$

33. $p(x) = -7x^6 + 4x - 8$

$\lim_{x\to-\infty} p(x) = -\infty$ **and** $\lim_{x\to\infty} p(x) = -\infty$

34.



$\lim_{x\to-\infty} p(x) = -\infty$ **and** $\lim_{x\to\infty} p(x) = \infty$

35. Sketch the graph of a polynomial function that could match statements $\lim_{x\to-\infty} p(x) = \infty$ and $\lim_{x\to\infty} p(x) = \infty$.