AP Precalculus

Name: Solutions Date: Period: 2A Review

Unit 2-A Review – Exponential and Logarithmic Functions

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets for lessons 2.1 – 2.8.

Find an equation that gives the *n*th term of each sequence. Instead of the initial value use the *k*th term of the sequence in your equation. *k* is given for each problem.

1. $\{116, 136, 156, 176, \ldots\}$ $k = 2$

+20 +20 +20

$d = 20$ $a_2 = 136$

$a_n = 136 + 20(n-2)$

2. $\left\{2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots\right\}$ $k = 1$

×1/3 ×1/3 ×1/3

$r = \frac{1}{3}$ $g_1 = 2$

$g_n = 2\left(\frac{1}{3}\right)^{n-1}$

3. For the sequence $\{8, 5, 2, -1, \ldots\}$, what is the 315th term?

-3

$a_n = 8 - 3(n-1)$

$a_{315} = 8 - 3(315-1) = -934$

A function has the following coordinate points. Could the function represent a linear function, exponential function, or neither?

4. $(3, 6), (4, 11), (5, 16)$

+5 +5

Linear

5. $(-10, 3), (-9, 6), (-8, 12)$

No → +3 +6

Yes ×2 ×2

Exponential

6. The following function is either linear or exponential. Which is it? Justify your answer.

+5 +5

x	6	11	16
$f(x)$	10	13	16

+3 +3

Linear because for each input change of 5, f changes at a constant rate 3.

Is each function linear or exponential. Identify the constant (slope or ratio) that causes the output values to change?

7. $y = -\left(\frac{3}{5}\right)^x$

Exponential
Ratio of $\frac{3}{5}$

8. $y - 7 = 6(x - 4)$

Linear
Slope of 6

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9. It is known that y is an exponential function and that it passes through the points $(3, 6)$ and $(9, 42)$. Write an equation for this function.

$y = a \cdot b^x$

$42 = 6 \cdot r^6$

$7 = r^6$

$\sqrt[6]{7} = r$

$y = 6 \cdot \left(\sqrt[6]{7}\right)^{x-3}$
or
$y = 42 \cdot \left(\sqrt[6]{7}\right)^{x-9}$

10. The value $4 \cdot 4 \cdot 4 \cdot 6$ is an output value of an exponential function of the form $f(x) = a \cdot b^x$, where a and b are constants. Write the function along with the input value that represents the output value.

$f(x) = 6 \cdot 4^x$

where $x =$ 3

11. Identify if the exponential function is growth or decay and justify your response. $f(x) = 1.4(0.07)^x$

Decay because $0 < b < 1$

12. Is the exponential function $f(x) = 8(1.7)^x$ increasing or decreasing?

Increasing

13. Is the exponential function $f(x) = -(6)^x$ concave up or concave down?

Concave down

14. Find $\lim_{x \to -\infty} f(x)$ for the function $f(x) = 4(0.3)^x$

$\lim_{x \to -\infty} f(x) = \infty$

Let $f(x)$ be a function on which a transformation occurs. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.

15. $f(x) = 9^x$ and $g(x) = f(x) \cdot 9$

$9^x \cdot 9 = 9^{x+1}$

Shift left 1 unit.

16. $f(x) = 2^x$ and $g(x) = \frac{f(x)}{16}$.

$2^x \cdot 2^{-4} = 2^{x-4}$

Shift right 4 units.

17. $f(x) = 7^x$ and $g(x) = \left(f(x)\right)^{-3}$

7^{-3x}

Horizontal dilation and reflection across the y-axis.

18. $f(x) = 7^x$ and $g(x) = -\frac{f(x)}{49}$

$-(7^x) \cdot 7^{-2} = -(7^{x-2})$

Reflection across the x-axis and shift right 2.

Evaluate the function at the given input values.

19. Let $h(x) = 6 \cdot 4^{x/3}$. Find $h(2)$

$6\sqrt[3]{16}$

20. Let $h(x) = 9 \cdot 2^{x/5}$. Find $h(-1)$

$\frac{9}{\sqrt[5]{2}}$

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21. Below is a table of values for an exponential function in the form $f(x) = a(b)^x + k$. Write the equation for this exponential function.

x	0	1	2	3	4
$f(x)$	3	6	18	66	258

3 → 12 → 48 → 192

b → ×4 ×4 ×4

$y = a \cdot 4^x + k$

$3 = a \cdot 4^0 + k$

$3 = a + k$

$3 = a + (6 - 4a)$

$3 = 6 - 3a$

$-3 = -3a \rightarrow a = 1$

$6 = a \cdot 4^1 + k$

$6 - 4a = k$

$6 - 4(1) = k$

$2 = k$

$f(x) = 4^x + 2$

22. The table gives the weight of a catfish at selected weeks t during its first eight weeks of life.

Week	1	3	6	8	9
Weight (pounds)	0.08	0.2	0.467	0.501	0.7

a. Use an exponential regression $f(t) = ab^t$ to model these data. Round to three decimals but store the original equation in your calculator.

$f(t) = 0.078(1.288)^t$

b. According to the model in your calculator, how much will the catfish weigh after 5 weeks?

0.275 pounds

c. According to the model in your calculator, how much will the catfish weigh after 7 weeks?

0.457 pounds

d. If the model continues past 9 weeks, when will the catfish weigh 1 pound?

$t = 10.098$. 10^{th} week

e. Use your model from part a to find the error at $t = 8$.

$f(8) = 0.591$

$0.591 - 0.501 = \mathbf{0.09}$

f. Is the value predicted an overestimate or underestimate of the actual value?

The predicted value 0.591 is an overestimate of the actual value of 0.501.

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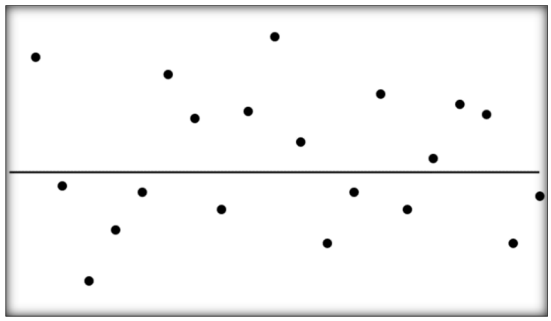
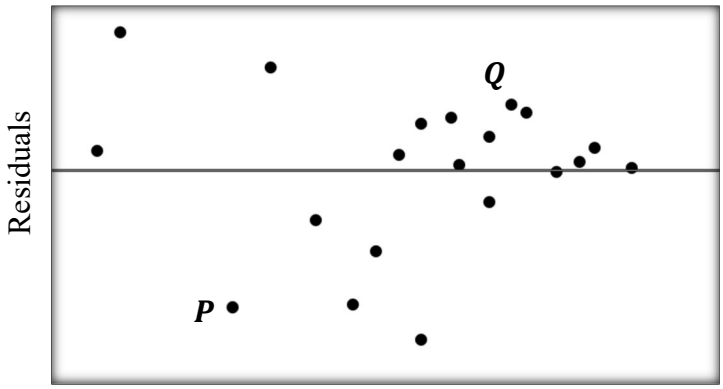
23. A student creates a model for a data set. The residual plot for their exponential regression is shown. Is the model appropriate? Why or why not?



Yes, because the residual plot has no clear pattern.

24. Several students' SAT scores are collected and compared to their GPA from their first year at college. A regression model was calculated with the SAT scores as input values and GPA from the first year at college as the output values. The residual plot for the regression model and data points is below.



a. The given residual plot has a point labeled P at the coordinate $(1050, -0.633)$. What does point P indicate in the context?

Because point P is below the x-axis, for the student with a 1050 SAT score, the model overestimates the student's GPA in their first year of college by 0.633.

b. The point labeled Q is at the coordinate $(1420, 0.3)$. What does point Q indicate in the context?

Because point Q is above the x-axis, for the student with a 1420 SAT score, the model underestimates the student's GPA in their first year of college by 0.3.

25. Use the tables to find the given values.

x	$g(x)$
-5	4
-2	10
0	-2
4	8

x	$f(x)$
-5	5
-2	9
0	0
4	-2

a. $g(f(4)) =$ $g(-2) =$ 10

b. $(f \circ g)(0) =$ $f(-2) =$ 9

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Let $f(x) = 3x^2 - 4x$ and $g(x) = 5 - 3x$.

26. Find $f(g(-1))$.

$g(-1) = 5+3$

$g(-1) = 8$

$f(8) = 3(8)^2 - 4(8)$

$192 - 32$

160

27. Find $(g \circ f)(1)$.

$f(1) = 3 - 4$

$f(1) = -1$

$g(-1) = 5+3$

8

28. Let $f(x) = 7x - 5$ and $h(x) = f(g(x))$. Fill in the table.

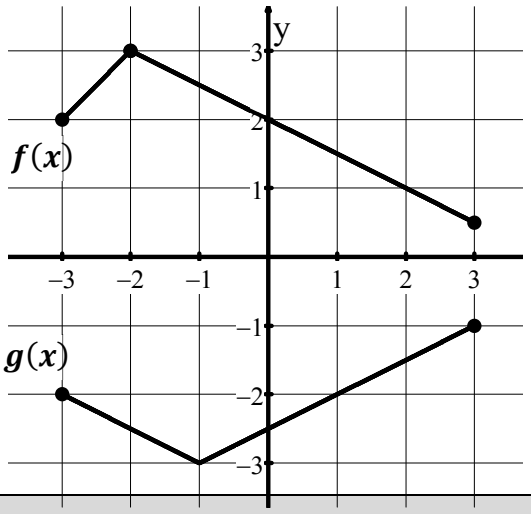
$f(g(-5)) = f(2) = 14 - 5 = 9$

$f(g(0)) = f(-1) = -7 - 5 = -12$

$f(g(5)) = f(3) = 21 - 5 = 16$

x	$g(x)$	$h(x)$
-5	2	9
0	-1	-12
5	3	16

Use the graph to find the following values.



29. Find $f(g(1))$. $f(-2) = 3$

30. Find $f(g(-3))$. $f(-2) = 3$

31. Find $g(f(2))$. $g(1) = -2$

Let $f(x) = \frac{1}{x-6}$ and $g(x) = 1 - x$.

32. Find $f \circ g$

$f(g(x)) = \frac{1}{(1-x)-6} = \frac{1}{-x-5}$

33. State the domain of $f \circ g$

$\mathbb{R}, x \neq -5$

34. Find $g \circ f$

$g(f(x)) = 1 - \frac{1}{x-6}$

35. State the domain of $g \circ f$

$\mathbb{R}, x \neq 6$

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36. Express h as a composition of two simpler functions f and g where $h(x) = f(g(x))$.

$$h(x) = \sqrt{7 - x^2}$$

$f(x) = \sqrt{x}$

$g(x) = 7 - x^2$

37. Find the inverse of $f(x) = (x-2)^2 + 1$ for $x \leq 2$. List the domain and range of $f^{-1}(x)$.

Domain of f: $x \leq 2$

Range of f: $y \geq 1$

$x = (y-2)^2 + 1$

$x - 1 = (y-2)^2$

$\pm\sqrt{x-1} = y - 2$

$f^{-1}(x) = -\sqrt{x-1} + 2$

Domain: $x \geq 1$

Range: $y \leq 2$

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