Name: Solutions Date:________________ Period: ______ **4A Review**

Unit 4A Review – Functions Involving Parameters, Vectors, and Matrices

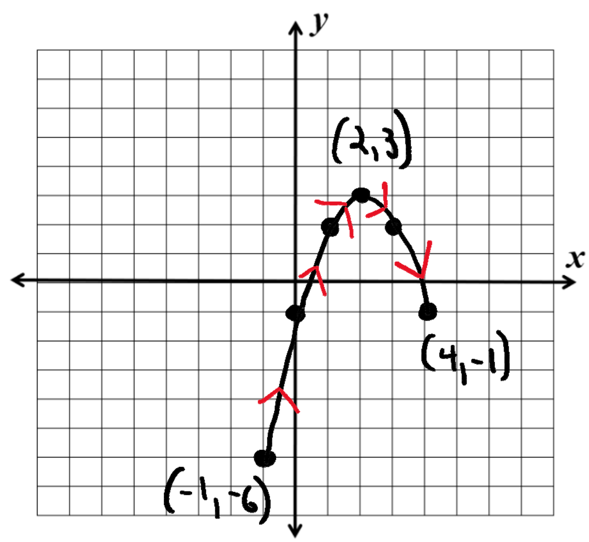
Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets for lessons 4.1 – 4.7.

1. Complete the table of numerical values for the given values of t using the parametric function $f(t) = (1 - t^2, 2t)$.

t	-3	-2	-1	0	1	2
x	-8	-3	0	1	0	-3
y	-6	-4	-2	0	2	4

2. An object is moving in the xy-plane so that at any time t, the position of the object can be found by evaluating the parametric equations $x(t) = t$ and $y(t) = -(t-2)^2 + 3$.

a. Graph the curve represented by the given parametric function for the restricted domain $-1 \le t \le 4$.



b. Find the horizontal relative extrema.

Horizontal relative **min** of -1 when $t = -1$
Horizontal relative **max** of 4 when $t = 4$

c. Find the vertical relative extrema.

Vertical relative **min** of -6 when $t = -1$
Vertical relative **max** of 3 when $t = 2$

d. Find the x-intercept(s). Show your work.

$y = 0$
$-(t-2)^2 + 3 = 0$
$(t-2)^2 = 3$
$t - 2 = \pm\sqrt{3}$
$t = 2 \pm \sqrt{3}$
$x = t$

x-intercepts $(2 \pm \sqrt{3}, 0)$ when $t = 2 \pm \sqrt{3}$

e. Find the y-intercept(s). Show your work.

$x = 0$
$t = 0$
$y = -(0-2)^2 + 3$
$y = -4 + 3 = -1$

y-intercept $(0, -1)$ when $t = 0$

3. A particles position for a given value of t can be found using the parametric equations $x(t) = t + 1$ and $y(t) = -t^2 + 4$, over the interval $-2 \le t \le 0$. Find the following.

a. The average rate of change of $x(t)$.

$$\frac{x(0) - x(-2)}{0 - -2} = \frac{1 - (-1)}{2} = \boxed{1}$$

b. The average rate of change of $y(t)$.

$$\frac{y(0) - y(-2)}{0 - -2} = \frac{4 - 0}{2} = \boxed{2}$$

c. The slope of the graph between the points on the graph that correspond to $t = -2$ and $t = 0$.

$t = -2 \rightarrow (-1, 0)$

$t = 0 \rightarrow (1, 4)$

$$\frac{4 - 0}{1 - -1} = \frac{4}{2} = \boxed{2}$$

4. Without the use of technology, determine which set of parametric equations will produce the same path as $f(t) = (t^2 + 2t + 1, t + 1)$, but will have a direction of particle motion in the opposite direction?

(A) $x(t) = -t^2 + 2t + 1,\ y(t) = -t + 1$

(B) $x(t) = t^2 - 2t + 1,\ y(t) = -t + 1$

(C) $x(t) = t^2 - 2t + 1,\ y(t) = -t - 1$

(D) $x(t) = t + 1,\ y(t) = t^2 + 2t + 1$

$x(-t) = t^2 - 2t + 1$

$y(-t) = -t + 1$

5. Find the parametric equations for the circle with the center at $(3, -2)$ and a radius of 2.

$x(t) = 3 + 2\cos t$

$y(t) = -2 + 2\sin t$

6. Find the parametric equations for the linear path of a particle that travels from the point $(-1, 3)$ to the point $(1, 7)$.

$(-1, 3)$ when $t = 0$

$(1, 7)$ when $t = 1$

$\frac{\Delta x}{\Delta t} = \frac{2}{1} = 2$

$\frac{\Delta y}{\Delta t} = \frac{4}{1} = 4$

$$\boxed{\begin{array}{l} x(t) = -1 + 2t \\ y(t) = 3 + 4t \end{array}}$$

7. Find the change of y with respect to x and determine how the two quantities in the implicitly defined function $15x^2 + y^2 - 20 = 0$ vary together on the interval $0 \le x \le 1$, and $y \ge 0$.

Find y when $x=0$

$y^2 - 20 = 0$

$y = \pm\sqrt{20} = \pm 2\sqrt{5}$

use $y = 2\sqrt{5}$ because of the restriction $y \ge 0$.

$(0, 2\sqrt{5})$

Find y when $x=1$

$15 + y^2 - 20 = 0$

$y^2 = 5$

$y = \pm\sqrt{5}$

$(1, \sqrt{5})$

$\frac{\Delta y}{\Delta x} = \frac{2\sqrt{5} - \sqrt{5}}{0-1} = \boxed{-\sqrt{5}}$

As one variable increases, the other decreases.

8. Find the vertex and orientation for the parabola given by $(x-2) = -\frac{2}{3}(y-4)^2$.

Vertex: $(2,4)$

opens left

9. Find the vertex and orientation for the parabola $x^2 - 2x + 2y + 3 = 0$. Show all your work to support your answer.

$x^2 - 2x = -2y - 3$

$x^2 - 2x + 1 = -2y - 3 + 1$

$(x-1)^2 = -2y - 2$

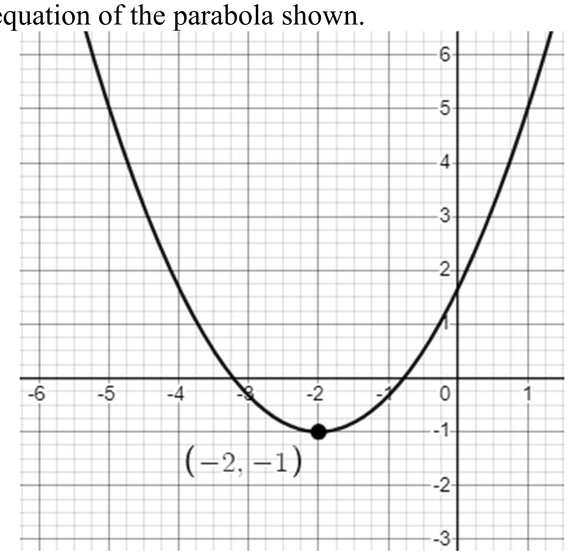
$(x-1)^2 = -2(y+1)$

$-\frac{1}{2}(x-1)^2 = (y+1)$

Vertex: $(1,-1)$

opens down

10. Which of the following is the equation of the parabola shown.



(A) $(x+1) = \frac{2}{3}(y+2)^2$

(B) $(x-1) = -\frac{2}{3}(y+2)^2$

(C) $(y+1) = \frac{2}{3}(x+2)^2$

(D) $(y-1) = -\frac{2}{3}(x-2)^2$

11. Use the equation $\frac{(x-2)^2}{4}+\frac{(y-5)^2}{16}=1$ to find the following of the ellipse.

a. center

$(2,5)$

b. horizontal or vertical

Vertical

c. vertices (ends of major axis)

$\sqrt{16}=4$

$5+4=9$

$5-4=1$

$(2,1)$ and $(2,9)$

d. ends of minor axis

$\sqrt{4}=2$

$2+2=4$

$2-2=0$

$(0,5)$ and $(4,5)$

e. foci

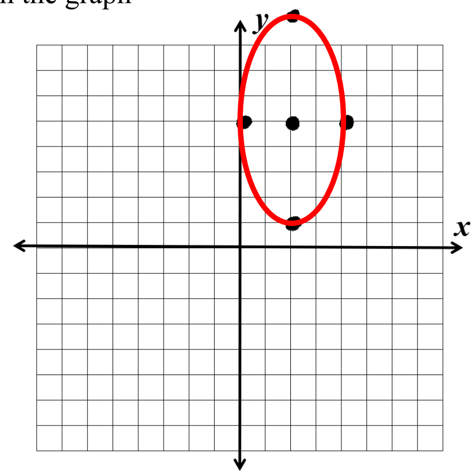
$16-4=c^2$

$12=c^2$

$\sqrt{12}=c$

$(2,\ 5\pm\sqrt{12})$

f. sketch the graph



12. Put the equation of an ellipse into standard form and then identify the center and orientation of the ellipse.
$x^2+25y^2-4x-50y+4=0.$

$x^2-4x+25(y^2-2y)=-4$

$x^2-4x+4+25(y^2-2y+1)=-4+4+25$

$(x-2)^2+25(y-1)^2=25$

$$\boxed{\frac{(x-2)^2}{25}+(y-1)^2=1}$$

Center: $(2,1)$

Orientation: horizontal

13. Find the parametrization of the conic given by $25(x+2)^2+2(y-5)^2=100.$

$\frac{25(x+2)^2}{100}+\frac{2(y-5)^2}{100}=\frac{100}{100}$

$\frac{(x+2)^2}{4}+\frac{(y-5)^2}{50}=1$ ellipse

$$\boxed{\begin{aligned}x(t)&=-2+2\cos t\\ y(t)&=5+\sqrt{50}\sin t\end{aligned}}$$

14. Use the equation $\frac{(x-1)^2}{16} - \frac{(y-3)^2}{4} = 1$ to find the following of the hyperbola.

a. center

$(1,3)$

b. horizontal/vertical

horizontal

c. Find the length of the transverse axis.

$2 \cdot \sqrt{16}$

8

d. Find the length of the conjugate axis.

$2 \cdot \sqrt{4}$

4

e. vertices

$1 + 4 = 5$

$1 - 4 = -3$

$\boxed{(-3,3) \quad (5,3)}$

f. foci

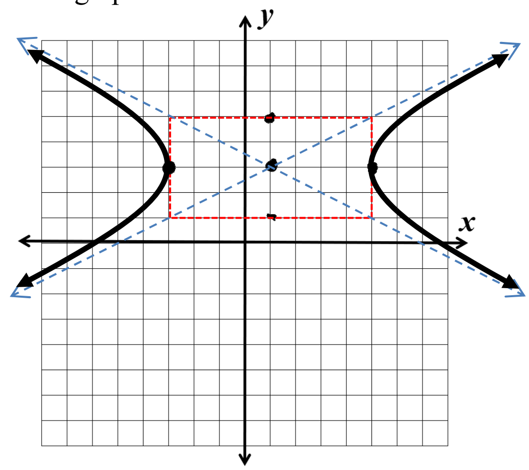
$16 + 4 = c^2$

$20 = c^2$

$\sqrt{20} = c$

$\boxed{(1 \pm \sqrt{20}, 3)}$

g. sketch the graph



15. Find the parametrization of the conic given by $\frac{(y-4)^2}{49} - \frac{(x-5)^2}{81} = 1$.

hyperbola

up/down

$x(t) = 5 + 9\tan t$

$y(t) = 4 + 7\sec t$

16. Which of the following represents the parametric equations $x(t) = t + 1, y(t) = 2t^2 + t + 1$, written in rectangular form?

(A) $y = 2x^2 + 4x + 3$

(B) $y = 2x^2 + 5x + 3$

(C) $y = 2x^2 - 4x + 2$

(D) $y = 2x^2 - 3x + 2$

$t = x - 1$

$y = 2(x-1)^2 + (x-1) + 1$

$y = 2(x^2 - 2x + 1) + x$

$y = 2x^2 - 4x + 2 + x$

$y = 2x^2 - 3x + 2$

17. Which of the following represents the parametrization of the conic $x^2 + 2x + 3 - y = 0$?

(A) $(t, (t+1)^2 + 2)$

(B) $(t, (t+1)^2 + 3)$

(C) $(t, (t-1)^2 + 2)$

(D) $(t, (t-1)^2 + 3)$

$y = x^2 + 2x + 3$

parabola

$y = x^2 + 2x + 1 + 3 - 1$

$y = (x+1)^2 + 2$