

Free Response Question #1

AP Precalculus

PRACTICE

Calculator Active – Answer the following practice FRQ #1 questions. Grade using the scoring rubric.

1. The function f is increasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x .

x	-2	-1	0	1	2
$f(x)$	2	6	18	54	162

The function g is given by $g(x) = 5.2(0.6)^x$.

A.

- i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(-2)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

$$h(-2) = g(f(-2)) = g(2) = 5.2(0.6)^2 = \mathbf{1.872}$$

From table $f(-2) = 2$

Value = 1 point

- ii. Find values of $f^{-1}(2)$, or indicate that it is not defined.

$$\text{From the table } f^{-1}(2) = -2$$

Value = 1 point

B.

- i. Find all values of x , as decimal approximations, for which $g(x) = 1$, or indicate there are no such values.

$$5.2(0.6)^x = 1$$

$$x = \mathbf{3.227}$$

Value = 1 point

- ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

$$\lim_{x \rightarrow \infty} g(x) = \mathbf{0}$$

End behavior with limit notation = 1 point

C.

- i. Based on the table, which of the following function types best models function f : linear, quadratic, exponential or logarithmic?

An exponential function best models f

Answer = 1 point

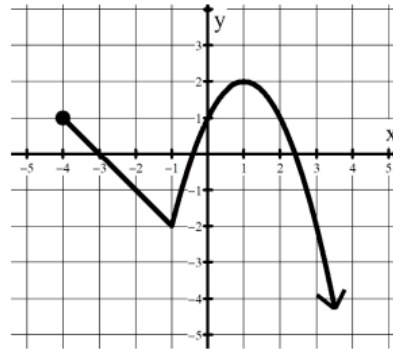
- ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f . Refer to the values in the table in your reasoning.

When the input increases by 1 unit, the output is multiplied by 3. Since equal changes in the input result in a constant ratio of 3 in the output, the function is exponential.

$$\frac{f(-1)}{f(-2)} = \frac{f(0)}{f(-1)} = \frac{f(1)}{f(0)} = \frac{f(2)}{f(1)} = \mathbf{3}$$

Reasoning = 1 point

2.



graph of f

The figure shows the graph of the function f on its domain of $x \geq -4$. The function g is given by $g(x) = 0.25x^3 - 3.2x^2 + 6$.

A.

- i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

$$h(1) = g(f(1)) = g(2) = 0.25(2)^3 - 3.2(2)^2 + 6 = -4.8 \quad \text{Value} = 1 \text{ point}$$

From graph $f(1) = 2$

- ii. Find all values of x for which $f(x) = -2$, or indicate there are no such values.

$$x = -1 \text{ and } 3 \quad \text{Values} = 1 \text{ point}$$

From the graph $f(-1) = -2$ and $f(3) = -2$

B.

- i. Find all values of x , as decimal approximations, for which $g(x) = -4$, or indicate there are no such values.

$$0.25x^3 - 3.2x^2 + 6 = -4 \quad \text{Values} = 1 \text{ point}$$

$$x = -1.663 \text{ and } 1.917 \text{ and } 12.545 \text{ (rounded } 12.546)$$

- ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

$$\lim_{x \rightarrow \infty} g(x) = \infty$$

End behavior with limit notation = 1 point

C.

- i. Determine if f has an inverse function.

f does not have an inverse on its domain of $x \geq -4$ Answer = 1 point

- ii. Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Three different input values, $x = -4, 0,$ and $2,$ produce the same output value 1 . Therefore, the function is not one-to-one and does not have an inverse.
 $f(-4) = f(0) = f(2) = 1$

Reason = 1 point

3. The function f is increasing and is defined for all real numbers. The table gives values for $f(x)$ at selected values of x .

x	-3	0	3	6	9
$f(x)$	2	6	10	14	18

The function g is given by $g(x) = 4.2 \ln(2x)$.

A.

- i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(3)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

$$h(3) = g(f(3)) = g(10) = 4.2 \ln(2(10)) = \mathbf{12.582}$$

From table $f(3) = 10$

Value = 1 point

- ii. Find values of $f^{-1}(6)$, or indicate that it is not defined.

From the table $f^{-1}(6) = 0$

Value = 1 point

B.

- i. Find all values of x , as decimal approximations, for which $g(x) = 3$, or indicate there are no such values.

$$4.2 \ln(2x) = 3$$

$$x = \mathbf{1.021}$$

Value = 1 point

- ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

$$\lim_{x \rightarrow \infty} g(x) = \infty$$

End behavior with limit notation = 1 point

C.

- i. Based on the table, which of the following function types best models function f : linear, quadratic, exponential or logarithmic?

A linear function best models f

Answer = 1 point

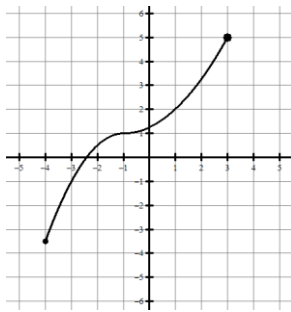
- ii. Give a reason for your answer in part C (i) based on the relationship between the change in the output values of f and the change in the input values of f . Refer to the values in the table in your reasoning.

When the input increases by 3 units, the output increases by 4 units. Since equal changes in the input produce equal changes in the output, the rate of change is constant at $\frac{4}{3}$, so the function is linear.

$$\frac{f(0)-f(-3)}{0-(-3)} = \frac{f(3)-f(0)}{3-0} = \frac{f(6)-f(3)}{6-3} = \frac{f(9)-f(6)}{9-6} = \frac{4}{3}$$

Reasoning = 1 point

4.



graph of f

The figure shows the graph of the function f on its domain of $-4 \leq x \leq 3$. The function g is given by $g(x) = -0.45x^2 - 5.2x - 2$.

A.

- i. The function h is defined by $h(x) = (g \circ f)(x) = g(f(x))$. Find the value of $h(1)$ as a decimal approximation, or indicate that it is not defined. Show the work that leads to your answer.

$$h(1) = g(f(1)) = g(2) = -0.45(2)^2 - 5.2(2) - 2 = -14.2 \quad \text{Value} = 1 \text{ point}$$

From graph $f(1) = 2$

- ii. Find all values of x for which $f(x) = -1$, or indicate there are no such values.

$$x = -3 \quad \text{Values} = 1 \text{ point}$$

From the graph $f(-3) = -1$

B.

- i. Find all values of x , as decimal approximations, for which $g(x) = 0$, or indicate there are no such values.

$$-0.45x^2 - 5.2x - 2 = 0 \quad \text{Values} = 1 \text{ point}$$

$$x = -11.157 \text{ and } x = -0.398$$

- ii. Determine the end behavior of g as x increases without bound. Express your answer using the mathematical notation of a limit.

$$\lim_{x \rightarrow \infty} g(x) = -\infty$$

End behavior with limit notation = 1 point

C.

- i. Determine if f has an inverse function. $-4 \leq x \leq 3$

f has an inverse on its domain of $-4 \leq x \leq 3$ Answer = 1 point

- ii. Give a reason for your answer based on the definition of a function and the graph of $y = f(x)$.

Every output value corresponds to exactly one input value. Therefore, the function is one-to-one and has an inverse function.

Reason = 1 point