Free Response Question #2

AP Precalculus

FRQ #2 Practice

Instructions: Answer each FRQ.

1)

t	0	9	45
Number of People	14	59	104

The table gives the number of people in the audience at Mr. Brust's Ted Talk. When he starts at 7 PM (t = 0), there are 14 people in the audience. Nine minutes later (t =9), there were 59 people in the audience. Forty-five minutes later (t = 45), there were 105 people in the audience.

The number of people in the audience can be modeled by the quadratic function P given by $P(t) = at^2 + bt + c$ where $P(t)$ is the number of people in the audience t minutes after the Ted Talk started.

(A) (i) Use the given data to write three equations that can be used to find the values for constants a, b, and c as decimal approximations.

1) $14 = a(0)^2 + b(0) + c$

2) $59 = a(9)^2 + b(9) + c$

3) $104 = a(45)^2 + b(45) + c$

(B)

(ii) Find the values of a, b, and c as decimal approximations.

$a = -0.083$

$b = 5.75$

$c = 14$

(C) (i) Use the given data to find the average rate of change in the number of people in the audience from t = 9 to t = 45 minutes. Express your answer as a decimal approximation. Show the computations that lead to your answer.

$$\frac{(f(45) - f(9))}{45 - 9} = 1.25$$

1.25 people per minute.

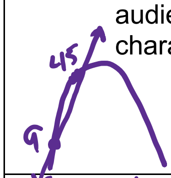
(ii) Use the average rate of change found in (i) to estimate the number of people in the audience at time t = 30 minutes. Show the computations that lead to your answer.

$y - y_1 = m(x - x_1)$

$y - 14 = 1.25(30 - 0)$

$y = 1.25(30) + 14 = \boxed{51.5 \text{ people}}$

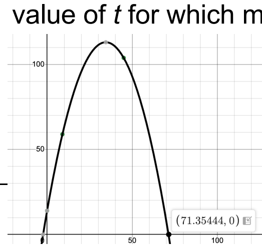
(iii) The average rate of change found in (i) can be used to estimate the number of people in the audience for times $t = d$ minutes, where $9 < d < 45$. Are these estimates less than or greater than the people in the audience predicted by P at time $t = d$ minutes. Explain your reasoning. Your explanation should include characteristics of the average rate of change and a reference to the graph of P.



The estimates are the secant line on the interval $9 < d < 45$. The graph of P is concave down on the interval so the estimates are less.

(D) The model P is an appropriate model to make predictions about the number of people in the audience if the predicted number of people in the audience is at least zero. Based on this information, what is the largest value of t for which model is appropriate? Give a reason for your answer.

(E)



P has a zero at 71.354 so the greatest value of t would be 71 minutes. as the values of $t > 71.354$ all result with $P < 0$.

2) Fans eagerly anticipated the new Algebros math video and so the first day (t = 0) that it was released on YouTube it had 85 thousand views. 45 days later (t = 45), there were 110 thousand views.

The number of views of the Algebros new video on a given day can be modeled by the function V given by $V(t) = a + b\ln(t+1)$, where *V(t)* is the number of views, in thousands, on day *t* since the initial day it was released.

(A) (i) Use the given data to write two equations that can be used to find the values for constants *a* and *b* in the expression for $V(t)$

1) $85 = a + b\ln(0+1)$ 2) $110 = a + b\ln(45+1)$

(ii) Find the values for *a* and *b* as decimal approximations.

$a = 85$ $b = 6.529$

(B) (i) Use the given data to find the average rate of change of number of views of the new Algebros Video, in thousands per day, from *t = 0* to *t = 45* days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

$$\frac{(f(45) - f(0))}{45 - 0}$$

$= 0.555555555556$

0.556 THOUSAND VIEWS per day.

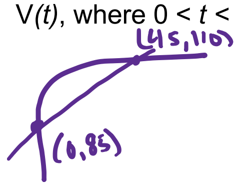
(ii) Use the average rate of change found in (i) to estimate the number of units of views, in thousands, on day *t = 25*. Show the work that leads to your answer.

$y - 85 = 0.556(25 - 0)$

$y = 0.556(25) + 85$

$y = 98.9$ thousand views.

(iii) Let A_t represent the estimate of the number of views of the new video, in thousands, using the average rate of change found in (i). For A_{25}, found in (ii), it can be show that $A_{25} < V(25)$. Explain why, in general, $A_t < V(t)$, where $0 < t < 45$.



A_t are y-coordinates on the secant line between $t=0$ and $t=45$. Since $V(t)$ is concave down on the interval the estimates will be less than the model.

(C) The Algebros reported that daily views of the new video decreased each day after *t = 45*. Explain why the error in model *V* increases after *t* = 45.

According to the model as t increases without bound the model will V(t) will increase so once the data goes against that by decreasing we know that V(t) will only work on the domain (0,45) for sure.

3) The temperature of TK's World's Best Chicken when it comes off the grill $(t = 0)$ is 165˚ Fahrenheit. Eight minutes later $(t = 8)$, TK finds the temperature of his World's Best Chicken is 120˚ Fahrenheit.

The temperature of TK's World's Best Chicken can be modeled by the function C given by $C(t) = ab^t + 68$, where $C(t)$ is the temperature of the World's Best Chicken, in degrees Fahrenheit (˚F), and t is the number of minutes since the soup was removed from the stove.

(A) (i) Use the given data to write two equations that can be used to find the values for constants a and b in the expression for C(t).

1) $165 = a \cdot b^0 + 68$ 2) $120 = a \cdot b^8 + 68$

(ii) Find the values of a and b as decimal approximations.

$a = 97$ $b = 0.925$

(B) (i) Use the given data to find the average rate of change of the temperature of TK's World's Best Chicken, in degrees Fahrenheit per minute, from $t = 0$ to $t = 8$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

$\frac{(f(8) - f(0))}{8 - 0} = -5.625$

-5.625 degrees per minute

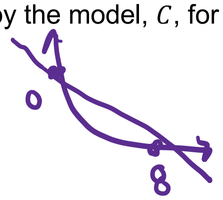
(ii) Use the average rate of change found in (i) to estimate the temperature of TK's World's Best Chicken when t = 6. Show the work that leads to your answer.

$y - 165 = -5.625(6 - 0)$

$y = -5.625(6) + 165$

$y = 131.25$ degrees.

(iii) The average rate of change calculated in part (i) can be used to estimate the temperature of TK's World's Best Chicken at time t for values of t between $t = 0$ and $t = 8$. Will the values estimated using the average rate of change be strictly greater than, strictly less than, or sometimes greater and sometimes less than the values predicted by the model, C, for $0 \leq t \leq 8$? Explain why this is the case.



The estimates will be greater than the model because the graph of C(t) is concave up on the interval $0 \leq t \leq 8$ which means the secant line will be above the model

(C) According to the model, the temperature of TK's Best Chicken will always exceed a certain temperature, R, which corresponds to the temperature of the room in which the soup is cooling. In other words, C$(t) > R$ for all t. However, TK finishes his TK's World's Best Chicken by the time the TK's World's Best Chicken's temperature has cooled to 15˚ Fahrenheit above the room temperature, R. Explain how this information could be used to determine an appropriate domain for C based on the context.

$y = 68$ is the horizontal asymptote the function is approaching. So we can add 15 to that and find when our model $C(t) = 68 + 15 = 83$. The x-coordinate will then tell us the highest value of our domain.