AP Precalc

Free Response Question #4

NOTES

Write your questions and thoughts here!

NO CALCULATOR – NO CONTEXT

Units 2 and 3 – Rewrite and Solve

Exponential, Logarithmic, Trig, Inverse Trig Functions

Directions:

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number. Angle measures for trigonometric functions are assumed to be in radians.
- Solutions to equations must be real numbers. Determine the exact value of any expression that can be obtained without a calculator. For example, $\log_2 8$, $\cos\left(\frac{\pi}{2}\right)$, and $\sin^{-1}(1)$ can be evaluated without a calculator.
- Unless otherwise specified, combine terms using algebraic methods and rules for exponents and logarithms, where applicable. For example, $2x + 3x$, $5^2 \cdot 5^3$, $\frac{x^5}{x^2}$, and $\ln 3 + \ln 5$ should be rewritten in equivalent forms.
- For each part of the question, show the work that leads to your answers.

NO REFERENCE SHEET ON THE EXAM!

Exponents	Logarithmic	Trigonometry
$\sqrt[3]{x^2} = x^{\frac{2}{3}}$ $x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$ $x^2 \cdot x^3 = x^5$ $\frac{x^2}{x^3} = x^{-1} = \frac{1}{x}$ $(x^3)^2 = x^6$	$a \log_b x = \log_b x^a$ $\log_b x + \log_b y = \log_b(xy)$ $\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$ $\log_b x = \frac{\log_c x}{\log_c b}$	$\csc x = \frac{1}{\sin x}$ $\tan x = \frac{\sin x}{\cos x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$ $\cot x = \frac{1}{\tan x}$ $\sin^{-1} x = \arcsin x$

Trig identities

Pythagorean:	Sum/Difference:	Double Angle:
$\sin^2 x + \cos^2 x = 1$ $\sin^2 x = 1 - \cos^2 x$ $\cos^2 x = 1 - \sin^2 x$ $1 + \cot^2 x = \csc^2 x$ $\tan^2 x + 1 = \sec^2 x$	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	$\sin(2\alpha) = 2\sin\alpha\cos\alpha$ $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$

Write your questions and thoughts here!

Part A: SOLVE

(A) The functions g and h are given by

$$g(x) = e^{(x-2)}$$

$$h(x) = 6\arccos\left(\frac{x}{4}\right)$$

(i) Solve $g(x) = 3$ for values of x in the domain of g.

(ii) Solve $h(x) = 5\pi$ for values of x in the domain of h.

Part B: REWRITE (include restricted domain)

(B) The functions j and k are given by

$$j(x) = \frac{\sin x}{\tan x} \qquad k(x) = \log_{10}(x-4) + \frac{1}{2}\log_{10}(x^4) - \log_{10}(4x)$$

(i) Rewrite $j(x)$ as a single term involving $\cos x$.

(ii) Rewrite $k(x)$ as a single logarithm base 10 without negative exponents in any part of the expression. Your result should be in the form of $\log_{10}(expression)$.

Part C: INPUT OUTPUT VALUES

(C) The function m is given by

$$m(x) = -2\sin(2x)$$

Find all input values in the domain of m that yield on output value of $\sqrt{3}$.

Exponent Rules

RULE	Rewrite using rational exponents.		
$\sqrt[3]{x^2} = x^{\frac{2}{3}}$	1. $\sqrt[5]{x^3}$	2. $\sqrt[3]{3^2}$	3. $\sqrt[3]{e^2}$

RULE	Rewrite without negative or rational exponents.		
$x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}$	1. $x^{\frac{1}{3}}$	2. 4^{-2}	3. $e^{\frac{1}{2}}$

RULE	Simplify.		
$x^2 \cdot x^3 = x^5$	1. $x^3 \cdot x^4$	2. $2^3 \cdot 2^x$	3. $e^{3x}e^{2x}$

RULE	Simplify.		
$\frac{x^2}{x^3} = x^{-1} = \frac{1}{x}$	1. $\frac{2^5}{2^7}$	2. $\frac{e^{3x}}{e^2}$	3. $\frac{x^4}{\sqrt{x}}$

RULE	Simplify.		
$(x^3)^2 = x^6$	1. $(2x^5)^3$	2. $(4^x)^2$	3. $\left(\sqrt{e}\right)^5$

Logarithmic Rules

RULE	Rewrite		
$a \log_b x = \log_b x^a$	1. $3 \log_5 x$	2. $2\log_7 e$	3. $\frac{1}{2} \ln 4$

Logarithmic Rules

RULE	Rewrite		
$\log_b x + \log_b y = \log_b xy$	1. $\log_4 x + \log_4 5$	2. $\log e^2 + \log y^3$	3. $5\ln 3 + \ln x$

RULE	Condense		
$\log_b x - \log_b y = \log_b \frac{x}{y}$	1. $\log_4(2x) - \log_4(5)$	2. $\log e^x - \log x^3$	3. $2\ln x - \ln x$

CHANGE OF BASE $\log_b x = \frac{\log_c x}{\log_c b}$	Change base to 3 1. $\log_2 x$	Change base to 2 2. $\log_8 7$	Change base to 10 3. $\log_3 100$

Trig Identities

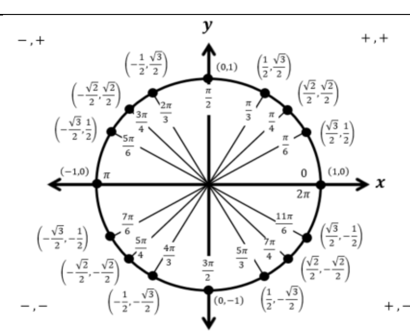
Pythagorean:	Sum/Difference:	Double Angle:
$\sin^2 x + \cos^2 x = 1$ $\sin^2 x = 1 - \cos^2 x$ $\cos^2 x = 1 - \sin^2 x$ $1 + \cot^2 x = \csc^2 x$ $\tan^2 x + 1 = \sec^2 x$	$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$ $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$	$\sin(2\alpha) = 2\sin\alpha\cos\alpha$ $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$ $= 1 - 2\sin^2\alpha$

TRIGONOMETRY

$$\csc x = \frac{1}{\sin x} \qquad \tan x = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

$$\cot x = \frac{1}{\tan x} \qquad \sin^{-1} x = \arcsin x$$



Free Response Question #4

AP Precalculus

PRACTICE

NO CALCULATOR – Answer the following practice FRQ #4 question. Grade using the scoring rubric.

1.

(A) The functions g and h are given by

$$g(x) = \log_3(2x - 5) \qquad h(x) = \sin^{-1}(6x)$$

(i) Solve $g(x) = 4$ for values of x in the domain of g.

(ii) Solve $h(x) = \frac{\pi}{4}$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = (\tan^2 x)(1 - \sin^2 x) \qquad k(x) = \frac{e^{3x}(e^{2x})}{(\sqrt{e})^{6x}}$$

(i) Rewrite $j(x)$ as a single term involving $\sin x$.

(ii) Rewrite $k(x)$ as a single exponent base e without negative exponents in any part of the expression. Your result should be in the form of $e^{(\text{expression})}$.

(C) The function m is given by

$$m(x) = 2\cos(2x) + 3$$

Find all input values in the domain of m that yield on output value of 4.

Your Score: ____ out of 6 points

Free Response Question #4

PRACTICE

AP Precalculus

NO CALCULATOR – Answer the following practice FRQ #4 question. Grade using the scoring rubric.

2.

(A) The functions g and h are given by

$$g(x) = e^{(x-2)} + e \qquad\qquad h(x) = 3\arccos(\pi x)$$

(i) Solve $g(x) = 2e$ for values of x in the domain of g.

(ii) Solve $h(x) = 2\pi$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \csc x - \sin x \qquad\qquad k(x) = 2\ln(4x^2) - \tfrac{1}{2}\ln(4x^4)$$

(i) Rewrite $j(x)$ as a product involving $\cot x$ and $\cos x$ and no other trigonometric functions.

(ii) Rewrite $k(x)$ as a single natural logarithm without negative exponents in any part of the expression. Your result should be in the form of $\ln(expression)$.

(C) The function m is given by

$$m(x) = 2\sin^2 x - \sin x$$

Find all input values in the domain of m that yield on output value of 0.

Your Score: ____ out of 6 points

Free Response Question #4

AP Precalculus

PRACTICE

NO CALCULATOR – Answer the following practice FRQ #4 question. Grade using the scoring rubric.

3.

(A) The functions g and h are given by

$$g(x) = \ln(x-5) + 4 \qquad h(x) = 4\arctan x$$

(i) Solve $g(x) = 6$ for values of x in the domain of g.

(ii) Solve $h(x) = -\pi$ for values of x in the domain of h.

(B) The functions j and k are given by

$$j(x) = \frac{\sin(2x)}{2\cos(x)} \qquad k(x) = \frac{(4^{3x})(8^{x})}{2}$$

(i) Rewrite $j(x)$ as a single term involving $\sin x$.

(ii) Rewrite $k(x)$ as a single exponent base 2 without negative exponents in any part of the expression. Your result should be in the form of $2^{(\text{expression})}$.

(C) The function m is given by

$$m(x) = 2e^{2x} - e^{x}$$

Find all input values in the domain of m that yield on output value of 0.

Your Score: ____ out of 6 points

Free Response Question #4

AP Precalculus

PRACTICE

NO CALCULATOR – Answer the following practice FRQ #4 question. Grade using the scoring rubric.

4.

(A) The functions g and h are given by

$$g(x) = 3^{(x-5)} \qquad h(x) = (2\cos x)(\sin x)$$

(i) Solve $g(x) = 9$ for values of x in the domain of g.

(ii) Solve $h(x) = \cos x$ for values of x in the interval of $[0, \pi]$.

(B) The functions j and k are given by

$$j(x) = (1 + \tan^2 x)(\cot^2 x) \qquad k(x) = \log_2(x+2) - 4\log_2(x) + \log_2(x^2)$$

(i) Rewrite $j(x)$ as a single term involving $\csc x$.

(ii) Rewrite $k(x)$ as a single logarithm base 2 without negative exponents in any part of the expression. Your result should be in the form of $\log_2(expression)$.

(C) The function m is given by

$$m(x) = \sin^{-1}(\cos x) + \frac{\pi}{2}$$

Find all input values in the domain of m that yield on output value of $\frac{2\pi}{3}$.

Your Score: ____ out of 6 points