11.1 Basic Identities and Trig Algebra

PRACTICE

Directions: Simplify to a single trig expression.

1) $\sin\beta \sec\beta$

$\sin B\left(\frac{1}{\cos B}\right) = \frac{\sin B}{\cos B}$

$\tan B$

* CHANGE ALL TO SIN/COS

2) $\tan\theta \csc\theta \cos\theta$

$\frac{\sin\theta}{\cos\theta}\left(\frac{1}{\sin\theta}\right)\left(\frac{\cos\theta}{1}\right)$

1

* CHANGE ALL TO SIN/COS

3) $\frac{\tan\alpha \cot\alpha}{\csc\alpha}$

$\frac{\frac{\sin\alpha}{\cos\alpha}\cdot\frac{\cos\alpha}{\sin\alpha}}{\frac{1}{\sin\alpha}}$

$= \frac{1}{\frac{1}{\sin\alpha}} = 1 \div \frac{1}{\sin\alpha}$

$= 1 \times \sin\alpha$

$= \sin\alpha$

* CHANGE TO SIN/COS

Directions: Verify the identity.

4) $\cos\theta \tan\theta \csc\theta = 1$

$\cos\theta\left(\frac{\sin\theta}{\cos\theta}\right)\frac{1}{\sin\theta} = 1$

$1 = 1$ ✓

* CHANGE ALL TO SIN/COS

5) $\cot x \sin x = \cos x$

$\frac{\cos x}{\sin x}\cdot\frac{\sin x}{1} = \cos x$

$\cos x = \cos x$ ✓

* CHANGE ALL TO SIN/COS

6) $\cot\mu \sec\mu \sin\mu = 1$

$\frac{\cos x}{\sin x}\cdot\frac{1}{\cos x}\cdot\sin x = 1$

$1 = 1$ ✓

* CHANGE ALL TO SIN/COS

7) $\frac{1+\sec\theta}{\tan\theta} = \cot\theta + \csc\theta$

$\frac{1}{\tan\theta} + \frac{\sec\theta}{\tan\theta} =$

$\cot\theta + \sec\left(\frac{1}{\tan\theta}\right) =$

$\cot\theta + \sec\theta(\cot\theta) =$

$\cot\theta + \frac{1}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta}\right) =$

$\cot\theta + \frac{1}{\sin\theta} =$

$\cot\theta + \csc\theta = \cot\theta + \csc\theta$ ✓

* SPLIT THE FIRST FRACTION

8) $\frac{\cot\theta}{\cos\theta} = \csc x$

$\cot\theta\left(\frac{1}{\cos\theta}\right) =$

$\frac{\cos\theta}{\sin\theta}\left(\frac{1}{\cos\theta}\right) =$

$\frac{1}{\sin\theta} =$

$\csc\theta = \csc\theta$ ✓

* CHANGE TO SIN/COS

9) $\frac{\frac{1}{\csc x}}{\frac{1}{\cot x}} = \cos x$

$\frac{1}{\csc x} \div \frac{1}{\cot x} =$

$\frac{1}{\csc x}\left(\frac{\cot x}{1}\right) =$

$\sin x\left(\frac{\cos x}{\sin x}\right) =$

$\cos x = \cos x$ ✓

* REWRITE THEN CHANGE TO SIN/COS

10) $\cot\mu(1+\sin\mu)=\cot\mu+\cos\mu$

$\cot\mu+\cot\mu\sin\mu=$

$\cot\mu+\frac{\cos\mu}{\sin\mu}(\sin\mu)=$

$\cot\mu+\cos\mu=\cot\mu+\cos\mu$

* DISTRIBUTE FIRST

11) $\tan x+\sec x=\frac{\sin x+1}{\cos x}$

$=\frac{\sin x}{\cos x}+\frac{1}{\cos x}$

$=\tan x+\sec x$

$\tan x+\sec x=\tan x+\sec x$

* SPLIT THE FRACTION ON THE RIGHT

12) $\frac{\frac{1}{\tan x}}{\frac{1}{\sec x}}=\csc x$

$\frac{1}{\tan x}\div\left(\frac{1}{\sec x}\right)=$

$\cot x\times(\sec x)=$

$\frac{\cos x}{\sin x}\left(\frac{1}{\cos x}\right)=$

$\frac{1}{\sin x}=\csc x$

$\csc x=\csc x$

* Rewrite as Multiplication FIRST!

13) $\tan\theta\left(\frac{\cos^2\theta+1}{\sin\theta}\right)=\cos\theta+\sec\theta$

$\frac{\sin\theta}{\cos\theta}\left(\frac{\cos^2\theta}{\sin\theta}+\frac{1}{\sin\theta}\right)=$

$\frac{\sin\theta}{\cos\theta}\left(\frac{\cos^2\theta}{\sin\theta}\right)+\frac{\sin\theta}{\cos\theta}\left(\frac{1}{\sin\theta}\right)$

$\frac{\cos^2\theta}{\cos\theta}+\frac{1}{\cos\theta}=$

$\cos\theta+\sec\theta=\cos\theta+\sec\theta$

* SPLIT FRACTION
* DISTRIBUTE

14) $\csc x+1=\sec x(\cot x+\cos x)$

$=\sec x\cot x+\sec x\cos x$

$=\frac{1}{\cos x}\left(\frac{\cos x}{\sin x}\right)+\frac{1}{\cos x}(\cos x)$

$=\frac{1}{\sin x}+1$

$\csc x+1=\csc x+1$

* DISTRIBUTE!

Directions: Multiply.

15) $\cot x(\sin x+\sec x)$

$\cot x\sin x+\cot x\sec x$

$\frac{\cos x}{\sin x}(\sin x)+\frac{\cos x}{\sin x}\left(\frac{1}{\cos x}\right)$

$\cos x+\frac{1}{\sin x}$

$\cos x+\csc x$

16) $(\csc\alpha-\sec\alpha)(\cos\alpha+\sin\alpha)$

$\csc\alpha\cos\alpha+\csc\alpha\sin\alpha-\sec\alpha\cos\alpha-\sec\alpha\sin\alpha$

$\frac{1}{\sin x}(\cos x)+\frac{1}{\sin x}(\sin x)-\frac{1}{\cos x}(\cos x)-\frac{1}{\cos x}(\sin x)$

$\cot x+1-1-\tan x$

$\cot x-\tan x$

17) $(\cos x-\sin x)(\cos x+\sin x)$

$\cos^2 x+\cos x\sin x-\cos x\sin x-\sin^2 x$

$\cos^2 x-\sin^2 x$

Directions: Factor.

18) $\cot^2 x - 3\cot x$

$\cot x(\cot x - 3)$

19) $2\csc^2 x - 5\csc x - 12$

X -24
+ -5

$(2\csc x - 8)(2\csc x + 3)$

$(\csc x - 4)(2\csc x + 3)$

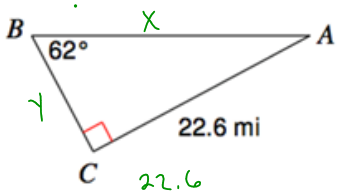
20) $\tan^2 x - \sec^2 x$

$(\tan x - \sec x)(\tan x + \sec x)$

→ DIFFERENCE OF SQUARES

Review: Solve each triangle. (find all sides and angles).

1)



$\angle A = 28°$

$\sin 62 = \frac{22.6}{X}$

$X = \frac{22.6}{\sin 62}$

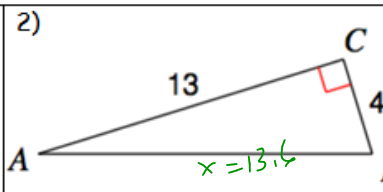
$X = 25.6$

$\tan 62 = \frac{22.6}{Y}$

$Y = \frac{22.6}{\tan 62}$

$Y = 12.0$

2)



$13^2 + 4^2 = x^2$

$169 + 16 = x^2$

$\sqrt{185} = \sqrt{x^2}$

$13.6 = x$

$\tan A = \frac{4}{13}$

$A = \tan^{-1}\left(\frac{4}{13}\right)$

$A = 17.1°$

$B = 72.9°$