11.2 Negative and Pythagorean Identities

PRACTICE

Directions: Simplify to a single trig expression.

1) $\sin(-\alpha)\cot(-\alpha)$

$-\sin\alpha(-\cot\alpha)$

$\cancel{\sin\alpha}\left(\frac{\cos\alpha}{\cancel{\sin\alpha}}\right)$

$\boxed{\cos\alpha}$

* change to sin/cos

2) $\frac{\tan(-u)\cot(-u)}{\csc(-u)}$

$= \frac{-\tan u \cdot -\cot u}{-\frac{1}{\sin u}}$

$= -\frac{\frac{\cancel{\sin u}}{\cancel{\cos u}}\left(\frac{\cancel{\cos u}}{\cancel{\sin u}}\right)}{\frac{1}{\sin u}}$

$= -\frac{1}{\frac{1}{\sin u}} = \boxed{-\sin u}$

*CHANGE TO SIN/COS

3) $\frac{(1-\cos x)(1+\cos x)}{\cos^2 x}$

$= \frac{1 + \cancel{\cos x} - \cancel{\cos x} - \cos^2 x}{\cos^2 x}$

$= \frac{1-\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

SPLIT THE FRACTION

$= \sec^2 x - 1$

$\boxed{= \tan^2 x}$

Directions: Verify the identity.

4) $\sin^3 x + \sin x\cos^2 x = \sin x$

$\sin x(\sin^2 x + \cos^2 x) = \sin x$

$\sin x(1) = \sin x$

$\sin x = \sin x$ ✓

*FACTOR OUT SINX

5) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$\cos^2 x + \sin^2 x = 1$

$1 = 1$ ✓

*CHANGE TO SIN/COS

6) $\frac{1+\tan\gamma}{1+\cot\gamma} = \tan\gamma$

Common denominator

$\frac{\left(\frac{\cos}{\cos}\right)1 + \frac{\sin\gamma}{\cos\gamma}}{\left(\frac{\sin}{\sin}\right)1 + \frac{\cos\gamma}{\sin\gamma}} = \frac{\frac{\cos\gamma+\sin\gamma}{\cos\gamma}}{\frac{\sin\gamma+\cos\gamma}{\sin\gamma}}$

$= \frac{\cancel{\cos\gamma+\sin\gamma}}{\cos\gamma}\left(\frac{\sin\gamma}{\cancel{\cos\gamma+\sin\gamma}}\right)$

$= \frac{\sin\gamma}{\cos\gamma} = \tan\gamma$

$\tan\gamma = \tan\gamma$ ✓

7) $\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x} = 2\sec x$

$\frac{\cos x(1-\sin x) + \cos x(1+\sin x)}{(1+\sin x)(1-\sin x)} = 2\sec x$

$\frac{\cos x \cancel{-\sin x\cos x} + \cos x + \cancel{\cos x\sin x}}{1 \cancel{-\sin x} \cancel{+\sin x} - \sin^2 x} =$

$\frac{2\cos x}{1-\sin^2 x} = 2\sec x$

$\frac{2\cancel{\cos x}}{\cos^{\cancel{2}} x} = 2\sec x$

$\frac{2}{\cos x} = 2\sec x$

*common DENOMINATOR

$2\sec x = 2\sec x$ ✓

8) $tan^2\,\theta\, cos^2\,\theta = 1 - cos^2\,\theta$

$\frac{\sin^2\theta}{\cancel{\cos^2\theta}} \cdot \cancel{\cos^2\theta} = 1 - \cos^2\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$1 - \cos^2\theta = 1 - \cos^2\theta$ ✓

* SUBSTITUTE $\sin^2\theta = 1 - \cos^2\theta$

9) $\frac{\sin A}{\csc A} + \frac{\cos A}{\sec A} = 1$

$\sin A\left(\frac{1}{\csc A}\right) + \cos A\left(\frac{1}{\sec A}\right) = 1$

$\sin A(\sin A) + \cos A(\cos A) = 1$

$\sin^2 A + \cos^2 A = 1$

$1 = 1$ ✓

* CHANGE TO SIN/COS

10) $cos^2\,\lambda + tan^2\lambda\, cos^2\lambda = 1$

$\cos^2\lambda(1 + \tan^2\lambda) = 1$

$\cos^2\lambda(\sec^2\lambda) = 1$

$\cancel{\cos^2\lambda}\left(\frac{1}{\cancel{\cos^2\lambda}}\right) = 1$

$1 = 1$

SUBSTITUTE $1 + \tan^2\lambda = \sec^2\lambda$

11) $\frac{\sec x - 1}{\sec x + 1} + \frac{\cos x - 1}{\cos x + 1} = 0$

$\frac{(\sec x - 1)(\cos x + 1) + (\cos x - 1)(\sec x + 1)}{(\sec x + 1)(\cos x + 1)} = 0$

$\frac{\sec x\cos x + \cancel{\sec x} - \cancel{\cos x} - 1 + \cos x\sec x + \cancel{\cos x} - \cancel{\sec x} - 1}{\sec x\cos x + \sec x + \cos x + 1} = 0$

$\frac{1 - 1 + 1 - 1}{1 + \sec x + \cos x + 1} = 0$

$\frac{0}{2 + \sec x + \cos x} = 0$

$0 = 0$

* COMMON DENOM.

12) $\sin\beta\,(\csc\beta - \sin\beta) = cos^2\,\beta$

$\sin\beta\csc\beta - \sin^2\beta = \cos^2\beta$

$\sin\beta\left(\frac{1}{\sin\beta}\right) - \sin^2\beta = \cos^2\beta$

$1 - \sin^2\beta = \cos^2\beta$

$\cos^2\beta = \cos^2\beta$

* SUBSTITUTE $1 - \sin^2\beta = \cos^2\beta$

13) $\cot x - \tan x = \frac{2\,cos^2\,x - 1}{\sin x \cos x}$

$\frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x} - \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} = \frac{2\cos^2 x - 1}{\sin x\cos x}$

$\frac{\cos^2 x - \sin^2 x}{\sin x\cos x} = \frac{2\cos^2 x - 1}{\sin x\cos x}$

$\frac{\cos^2 x - (1 - \cos^2 x)}{\sin x\cos x} =$

* SUBSTITUTE $1 - \cos^2 x = \sin^2 x$

$\frac{\cos^2 x - 1 + \cos^2 x}{\sin x\cos x} = ?\ \frac{2\cos^2 x - 1}{\sin x\cos x}$ ✓

14) $\sin^2 x\cos^3 x = (\sin^2 x - \sin^4 x)(\cos x)$

$= \sin^2 x(1-\sin^2 x)(\cos x)$

$= \sin^2 x(\cos^2 x)(\cos x)$

$= \sin^2 x\cos^3 x$

$*\cos^2 x = 1-\sin^2 x$

15) $\cos^2 x = \dfrac{\csc x\cos x}{\tan x+\cot x}$

$$\cos^2 x = \frac{\frac{1}{\sin x}\left(\frac{\cos x}{1}\right)}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}$$

$$= \frac{\frac{\cos x}{\sin x}(\sin x)(\cos x)}{\frac{\sin x}{\cos}+\frac{\cos x}{\sin x}(\sin x)(\cos x)}$$

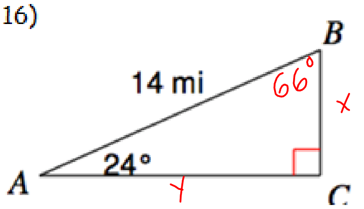
$$= \frac{\cos^2 x}{\sin^2 x+\cos^2 x}$$

$$= \frac{\cos^2 x}{1}$$

$\cos^2 x = \cos^2 x$

Review: Solve each triangle. (find all sides and angles).

16)



$\sin 24 = \frac{x}{14}$

$14\cdot\sin 24 = x$

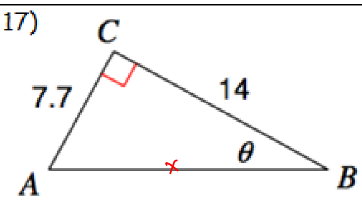
$\boxed{5.69 = x}$

$\cos 24 = \frac{y}{14}$

$14\cdot\cos 24 = y$

$\boxed{12.79 = y}$

17)



$\tan A = \frac{14}{7.7}$

$\boxed{\angle A = 61.2^\circ}$

$\angle B = 90 - 61.2$

$\boxed{\angle B = 28.8}$

$x^2 = 7.7^2 + 14^2$

$\sqrt{x^2} = \sqrt{255.29}$

$\boxed{x = 15.98}$