11.3 Sum and Difference Identities

PRACTICE

Directions: Tell whether each statement is true or false.

1) $\sin 75 = \sin 50 \cos 25 - \cos 25 \sin 25$ FALSE	2) $\cos 15 = \cos 60 \cos 45 + \sin 60 \sin 45$ TRUE	3) $\tan 225 = \dfrac{\tan 180 - \tan 45}{1 + \tan 180 \tan 45}$ FALSE

Directions: Write the expression as the sine, cosine or tangent of an angle.

4) $\sin 42 \cos 17 - \cos 42 \sin 17$ $\sin 25^\circ$	5) $\dfrac{\tan 19 + \tan 47}{1 - \tan 19 \tan 47}$ $\tan 66^\circ$	6) $\cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$ $\frac{4\pi}{12}$, $\frac{3\pi}{12}$, $\frac{4\pi}{12}$, $\frac{3\pi}{12}$ $\frac{4\pi}{12} - \frac{3\pi}{12}$ $\boxed{\cos\frac{\pi}{12}}$

Directions: Use the sum or difference identity to find the exact value.

7) $\tan 195^\circ$ $\quad 150 + 45$

$$\frac{\tan 150 + \tan 45}{1 - \tan 150 \tan 45} = \frac{3\left(-\frac{\sqrt{3}}{3} + 1\right)}{3\left(1 - -\frac{\sqrt{3}}{3}(1)\right)}$$

$$= \frac{-\sqrt{3} + 3}{3 + \sqrt{3}} \cdot \frac{(3 - \sqrt{3})}{(3 - \sqrt{3})} = \frac{-3\sqrt{3} + 3 + 9 - 3\sqrt{3}}{9 - 3}$$

$$= \frac{12 - 6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$$

8) $\cos 255^\circ$ $\quad 300 - 45$

$$= \cos 300 \cos 45 + \sin 300 \sin 45$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2}\right) + \frac{-\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{-\sqrt{6}}{4}$$

$$\boxed{= \frac{\sqrt{2} - \sqrt{6}}{4}}$$

9) $\sin 165^\circ$ $\quad 135 + 30$

$$= \sin 135 (\cos 30) + \cos 135 (\sin 30)$$

$$= \frac{\sqrt{2}}{2}\left(\frac{\sqrt{3}}{2}\right) + \frac{-\sqrt{2}}{2}\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{6}}{4} + \frac{-\sqrt{2}}{4}$$

$$\boxed{= \frac{\sqrt{6} - \sqrt{2}}{4}}$$

10) $\cos\frac{13\pi}{12}$ $\quad \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}$

$$\cos\frac{3\pi}{4} \cdot \cos\frac{\pi}{3} - \sin\frac{3\pi}{4} \cdot \sin\frac{\pi}{3}$$

$$= \frac{-\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$\boxed{= \frac{-\sqrt{2} - \sqrt{6}}{4}}$$

11) $\sin\frac{5\pi}{12}$ $\quad \frac{8\pi}{12} - \frac{3\pi}{12} = \frac{2\pi}{3} - \frac{\pi}{4}$

$\sin\left(\frac{2\pi}{3}\right)\cos\frac{\pi}{4} - \cos\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{\sqrt{6}}{4} - \left(\frac{-\sqrt{2}}{4}\right)$

$\boxed{= \frac{\sqrt{6}+\sqrt{2}}{4}}$

12) $\tan\frac{\pi}{12}$ $\quad \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

$= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{\pi}{3}\tan\frac{\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}(1)} =$

$= \frac{\sqrt{3}-1}{1+\sqrt{3}}\frac{(1-\sqrt{3})}{(1-\sqrt{3})} = \frac{\sqrt{3}-3-1+\sqrt{3}}{1-3}$

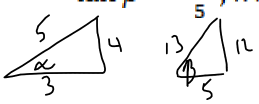
$= \frac{2\sqrt{3}-4}{-2} \quad = \frac{2\sqrt{3}}{-2} + \frac{-4}{-2} \quad = \boxed{-\sqrt{3}+2}$

Directions: Find the exact value.

13) $\sin(\alpha - \beta)$

Given: $\cos\alpha = \frac{3}{5}$, *where* $0 < \alpha < \frac{\pi}{2}$

$\tan\beta = \frac{12}{5}$, *where* $0 < \beta < \frac{\pi}{2}$



$\sin\alpha\cos\beta - \cos\alpha\sin\beta$

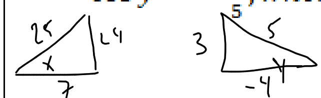
$= \frac{4}{5}\left(\frac{5}{13}\right) - \frac{3}{5}\left(\frac{12}{13}\right)$

$= \frac{20}{65} - \frac{36}{65} = \boxed{\frac{-16}{65}}$

14) $\tan(x - y)$

Given: $\cos x = \frac{7}{25}$, *where* $0° < x < 90°$

$\cos y = -\frac{4}{5}$, *where* $90° < y < 180°$



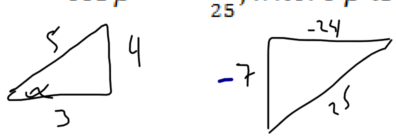
$= \frac{\left(\frac{24}{7}\right) - \left(\frac{-3}{4}\right)}{1+\left(\frac{24}{7}\right)\left(\frac{-3}{4}\right)} = \frac{\frac{117}{28}}{1+\left(\frac{-72}{28}\right)} = \frac{\frac{117}{28}}{\frac{-44}{28}}$

$= \frac{117}{28}\left(\frac{28}{-44}\right) = \boxed{-\frac{117}{44}}$

15) $\sin(\alpha + \beta)$

Given: $\sin\alpha = \frac{4}{5}$, *where* α *is in Quadrant I*

$\cos\beta = -\frac{24}{25}$, *where* β *is in Quadrant III*



$\sin\alpha\cos\beta + \cos\alpha\sin\beta$

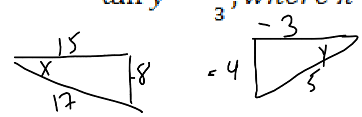
$= \frac{4}{5}\left(\frac{-24}{25}\right) + \left(\frac{3}{5}\right)\left(\frac{-7}{25}\right)$

$= \frac{-96}{125} + \frac{-21}{125} = \boxed{\frac{-117}{125}}$

16) $\cos(x + y)$

Given: $\cos x = \frac{15}{17}$, *where* $\frac{3\pi}{2} < x < 2\pi$

$\tan y = \frac{4}{3}$, *where* $\pi < y < \frac{3\pi}{2}$



$\cos x\cos y - \sin x\sin y$

$= \left(\frac{15}{17}\right)\left(\frac{-3}{5}\right) - \frac{-8}{17}\left(\frac{-4}{5}\right)$

$= \frac{-45}{85} - \frac{32}{85} = \boxed{\frac{-77}{85}}$

Directions: Is the equation an identity? Explain using the sum or difference identities

17) $\cos(x - \pi) = \cos x$

$\cos x \cos \pi + \sin x \sin \pi = \cos x$

$\cos x(-1) + \sin x(0) = \cos x$

$-\cos x = \cos x$

NO ITS NOT AN IDENTITY

18) $\sin(x - \pi) = \sin x$

$\sin x \cos \pi - \cos x \sin \pi = \sin x$

$= \sin x \cdot -1 - \cos x \cdot 0 = \sin x$

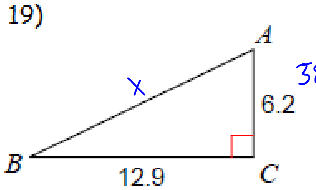
$= -\sin x - 0 = \sin x$

$= -\sin x \neq \sin x$

NOT AN IDENTITY

Directions: Solve each triangle.

19)



$6.2^2 + 12.9^2 = x^2$

$38.44 + 166.41 = x^2$

$\sqrt{204.85} = \sqrt{x^2}$

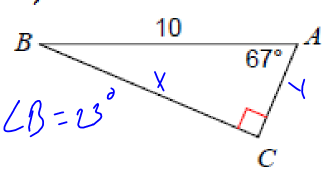
$\boxed{14.3 = x}$

$\tan B = \frac{6.2}{12.9}$

$\boxed{\angle B = 25.7^0}$

$\boxed{\angle A = 64.3^0}$

20)



$\angle B = 23^0$

$\sin 67 = \frac{x}{10}$

$\boxed{9.2 = x}$

$\cos 67 = \frac{y}{10}$

$\boxed{3.9 = y}$