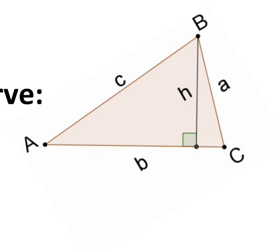
12.1 Law of Sines

Bean Lake, Marshall, Missouri:

While out hiking one day, Mr. Bean and Mr. Brust get lost and find themselves on opposite sides of Bean Lake. Bean wants to swim to Brust, but is afraid the distance is too far. Mr. Brust is a trig wiz and estimates the angle Mr. Bean is from the opposite shoreline to be 88°. He then paces 100 yards down the shoreline, and measures the angle Mr. Bean is to the shoreline where he currently standing to be 44°.

Brust plans on using the ***Law of Sines*** to estimate the width of the lake for Mr. Bean.

Lets observe:



Law of Sines

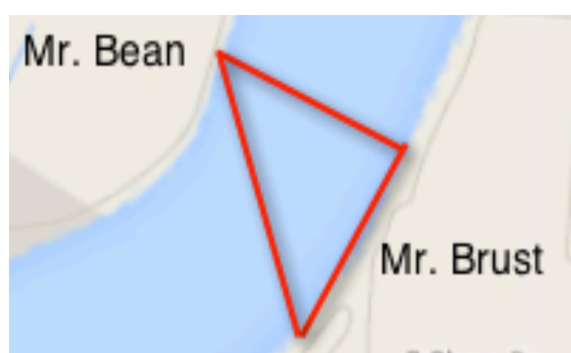
Let ΔABC be any triangle with *a*, *b*, and *c* representing the measures of the sides opposite the angles with measurements A, B and C respectively. Then the following is true:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

You can use the Law of Sines to solve a triangle when you are given the following information:

1. Two angle measures and any side length. (AAS or ASA information)
2. Two side lengths and the measure of a non-included angle (SSA information)

Label the triangle with Brust's measurements. Use the *Law of Sines* to find the width of the lake.



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Solving Triangles

Solving Triangles means finding the measure of every side and every angle. Sometimes this is as easy as the first example. This isn't always the case, however.
When the measure of two sides of a triangle and the measure of the angle opposite one of them is given (In Geo: SSA), there may not always be a solution. In fact, there could be:

1. No solution
2. Exactly one Solution
3. Two unique Solutions

Saw WHAT?!?! 2 possible triangles? Let's throw in some conceptual elixir....

Suppose we know the measures of sides a and b and ∡ A. Two cases exist:

Case #1. ∡A is an ***obtuse*** angle. There are 2 possible situations:

$a \leq b$ or $a > b$

This is called The Ambiguous Case!

Case #2. ∡A is an acute angle. There are 4 possible situations. To consider them, we will compare side a to the value of $b \sin A$

If $a = b \sin A$, ______________________

If $a < b \sin A$, ______________________

If $a > b \sin A$ and $a \geq b$, ______________________

If $a > b \sin A$ and $a < b$, ______________________ Write your questions and thoughts here!

12.1 Law of Sines

*Solve means find the value of every side and every angle.

Examples Determine the number of solutions. When possible, solve the triangles that exist.

Solve* **ΔABC** if A = 63°, b = 18 and a = 17

Solve **ΔABC** if a = 32, c = 20 and A = 112°

You try!

Solve **ΔABC** if b = 15, c = 13 and C = 50°

Solve **ΔABC** if a = 33, b = 50 and A = 132°

Now summarize what you learned!

Skillz Review

Important Note: (*sin x*)(*sin x*) = (*sin x*)^2 = *sin*^{2} *x*

$\frac{2}{3}+\frac{1}{4}=$	$\frac{2x}{3}+\frac{x}{4}=$	$\frac{2\sin x}{3}+\frac{\sin x}{4}=$
$\left(\frac{2}{3}\right)^2=$	$\left(\frac{2x}{3}\right)^2=$	$\left(\frac{2\sin x}{3}\right)^2=$

Algebra 2

Law of Sines Practice

State the number of possible triangles that can be formed using the given measurements. Ambiguous Case?

1) $m\angle A = 144°$, $c = 24$ km, $a = 4$ km

2) $m\angle A = 60°$, $c = 34$ mi, $a = 33$ mi

3) In $\triangle BCA$, $m\angle B = 110°$, $a = 13$ yd, $b = 30$ yd

4) In $\triangle EFD$, $m\angle E = 71°$, $d = 34$ m, $e = 33$ m

Solve each triangle. Round your answers to the nearest tenth.

5) $m\angle A = 82°$, $a = 19$ km, $c = 9$ km

6) $m\angle C = 22°$, $b = 17$ yd, $c = 12$ yd

7) $m\angle C = 70°$, $m\angle B = 22°$, $a = 16$ m

8) $m\angle C = 130°$, $b = 14$ m, $c = 30$ m

9) In $\triangle CAB$, $m\angle C = 73°$, $b = 30$ in, $c = 26$ in

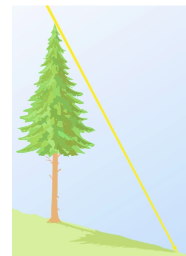
10) In $\triangle XYZ$, $m\angle X = 39°$, $z = 30$ m, $x = 28$ m

11) In $\triangle RPQ$, $m\angle R = 82°$, $m\angle Q = 43°$, $q = 20$ km

12) In $\triangle STR$, $m\angle S = 155°$, $r = 25$ mi, $s = 25$ mi

1. Solve **ΔABC** if b = 15, c = 13 and C = 50°

2. A tree growing on the side of a hill casts a 102-foot shadow straight down the hill (see figure). Find the vertical height of the tree if, relative to the horizontal, the hill slopes 15° and the angle of elevation of the sun is 62°.



3. 3 Algebros are camping in the woods: Bean, Brust and Sully. They each have their own tent and the tents are set up in a triangle. Bean and Brust are 10m apart. The angle formed at Bean is 30°. The angle formed at Sully is 105°. How far apart are Brust and Sully?

4. In order to reach the top of a hill that is 250 feet high, one must travel 2000 feet up a road that leads to the top. Find the number of degrees contained in the angle that the road makes with the horizontal.

5. Two markers A and B are on the same side of a river are 58 feet apart. A third marker is located across the river at point C. A surveyor determines that ∢CAB=68° and ∢ABC=52°.

a) What is the distance between points A and C?

b) What is the distance across the river?