

12.3 Area of Triangles

The side lengths of a triangular traffic sign are approximately 27 cm, 27 cm and ????. The angle between the given legs is 40° . Approximate the area of the sign. Round your answer to the nearest whole cm^2 .

Area of Triangles

Let $\triangle ABC$ be any triangle with a , b , and c representing the measures of the sides opposite the angles with measurements A , B and C respectively. Then the area of the triangle equals:

Hero's (Heron's) Formula

$$A = \frac{1}{2} ab \sin C$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where s is the *semiperimeter* of the triangle.

Example 1. Find the area of $\triangle ABC$ if $a = 15$ ft., $b = 18$ ft. and $m\angle C = 100^\circ$.

Example 2. Find the area of $\triangle ABC$ if $a = 20$, $b = 40$ and $c = 30$.

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You try! Find the following areas:

- Find the area of $\triangle RST$ if $r = 120$, $s = 80$ and $t = 115$.
- Find the area of $\triangle XYZ$ if $y = 6$ m, $m\angle Z = 127^\circ$ and $z = 9$ m.
- The adjacent sides of a parallelogram measure 8 cm and 12 cm and one angle between them measures 60° . Find the area of the parallelogram.

Now
summarize
what you
learned!

Skillz Review

Important Note: $(\sin x)(\sin x) = (\sin x)^2 = \sin^2 x$

$$(x + 3)(2x - 1)$$

$$(\sin x + 3)(2\sin x - 1)$$

$$(\sin x + \cos x)(2\sin x - \cos x)$$

$$\left(\frac{12}{5}\right)\left(\frac{5}{4}\right) =$$

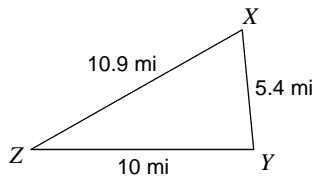
$$\left(\frac{12}{\tan x}\right)\left(\frac{\tan x}{4}\right) =$$

$$\left(\frac{12 \sin x}{\tan x}\right)\left(\frac{\tan x}{4 \cos x}\right) =$$

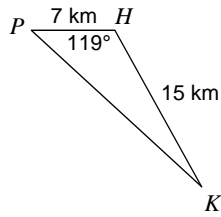
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Find the area of each triangle to the nearest tenth.

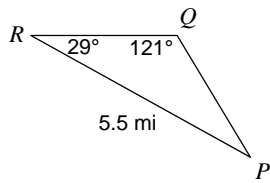
1)



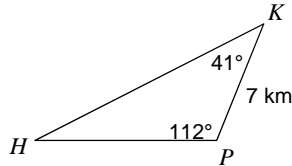
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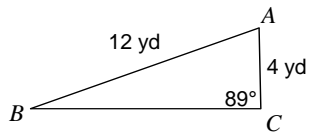
3)



4)



5)



6) In $\triangle QRP$, $q = 5$ m, $m\angle R = 56^\circ$, $m\angle Q = 54^\circ$

7) In $\triangle RST$, $m\angle S = 56^\circ$, $m\angle R = 56^\circ$, $t = 4$ cm

8) In $\triangle HPK$, $p = 13$ m, $k = 18$ m, $h = 16.6$ m

9) In $\triangle DEF$, $f = 8$ ft, $m\angle D = 106^\circ$, $d = 12$ ft

10) In $\triangle TRS$, $r = 16$ cm, $s = 7.2$ cm, $m\angle T = 68^\circ$

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APPLICATION 12.3

1. Find the area of $\triangle ABC$ if $b = 15$, $c = 13$ and $C = 50^\circ$
2. A triangular parcel of land has sides of length 680 feet, 320 feet, and 802 feet. What is the area of the parcel of land? If land is valued at \$2100 per acre (1 acre is 43560 square feet), what is the value of the parcel of land.
3. Farmer Bean has a triangular field with sides of 240 feet, 300 feet, and 360 feet. Through experimentation based off of past practice with similar compounds, he finds that Mr. Brust's videos work as great fertilizer! He wants to apply fertilizer to the field. If one 40-pound bag of Brust's videos can fertilize 6,000 square feet, how many bags must he buy to cover the field?
4. **Calculator Programming.** For this Application Problem, you will need a TI83 or Ti84 calculator! When computing several different triangle areas, it may be best to write a small program. This is actually easier than you might think! We will start by naming our program: HERO. Use the following keystrokes:

Step 1 (**Naming the Program**):

`PRGM` `▶▶` `ENTER` `^` `SIN` `×` `7` `ENTER`

This has essentially created a new program with the name "HERO." We know that Hero's formula is :

Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$ where s is the **semiperimeter** of the triangle.

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The calculator needs to know the lengths of the three sides a, b, and c. We will ask the program in the I/O (input/output) menu to prompt for the side lengths a, b and c:

Step 2 (**Prompting for a, b, c**): `PRGM ▶ 2 ALPHA MATH , ALPHA APPS , ALPHA PRGM ENTER`

We will now have the program calculate the semiperimeter, and store the value into the variable s:

Step 3 (**Calculating the Semiperimeter**):

`(1 ÷ 2) (ALPHA MATH + ALPHA APPS + ALPHA PRGM) STO ALPHA LN ENTER`

We are almost done! Before we calculate the area, we will ask the program to output some friendly words:

Step 4 (**Customizing the output**): `PRGM ▶ 3 2nd ALPHA + 4 ^ SIN 0 MATH × SIN MATH 0 x2 LN + ENTER`

And finally, we can now calculate the area:

Step 5 (**Calculating the Area**):

`2nd x2 ALPHA LN (ALPHA LN - ALPHA MATH) (ALPHA LN - ALPHA APPS) (ALPHA LN - ALPHA PRGM) ENTER`

When finished, your program should look like this →

It is now time to run (or execute) our program!

Return to the home screen: `2nd MODE`

```
PROGRAM:HERO
:Prompt. A,B,C
:(1/2)(A+B+C)→S
:Disp "THE AREA
IS"
:√(S(S-A)(S-B)(S
-C)
:█
```

Now, test your program on a triangle with the following side lengths: a = 10, b = 12 and c = 13.

Step 6 (**Running your program**): `PRGM ENTER ENTER 1 0 ENTER 1 2 ENTER 1 3 ENTER`

Your program should give you the correct area if you programmed the calculator correctly:

```
A=?10
B=?12
C=?13
THE AREA IS
      56.99506558
```

Congratulations!! You may now use your program to find the areas of the triangles with the following side lengths:

- | | | | | | | | |
|----|--------|----|--------|----|---------|----|----------|
| a. | a = 14 | b. | a = 54 | c. | d = .09 | d. | x = 1100 |
| | b = 12 | | b = 67 | | e = .04 | | y = 103 |
| | c = 5 | | c = 90 | | f = .1 | | z = 1202 |