Practice 14.2

For each sequence, state if it is arithmetic, geometric, or neither.If it is arithmetic, tell the common difference. If it is geometric, tell the common ratio. If it is neither, chill out and move on to the next problem.

1) $-1,\ 6,\ -36,\ 216,\ -1296,\ \ldots$

Geometric, Common Ratio = -6

2) $11,\ -9,\ -29,\ -49,\ -69,\ \ldots$

Arithmetic, Common Difference = -20

3) $2,\ \frac{5}{2},\ 3,\ \frac{7}{2},\ 4,\ \ldots$

Arithmetic, Common Difference = 0.5 (or 1/2)

4) $-6,\ 24,\ -126,\ 624,\ -3126,\ \ldots$

Neither

5) $32,\ 36,\ 40,\ 44,\ 48,\ \ldots$

Arithmetic, Common Difference = 4

6) $0.4,\ 2,\ 10,\ 50,\ 250,\ \ldots$

Geometric, Common Ratio = 5

7) $a_n = -\frac{19}{24} + \frac{5}{3}n$

Arithmetic, Common Difference = (5/3)

8) $a_n = 8 + 6n$

Arithmetic, Common Difference = 6

9) $a_n = 3 \cdot (-6)^{n-1}$

Geometric, Common Ratio = -6

10) $a_n = \frac{2n}{2n+1}$

Neither

Determine if the sequence is arithmetic. If it is, find the common difference, the term named in the problem, and the explicit formula.

11) $10,\ 16,\ 22,\ 28,\ \ldots$
Find a_{25}

Arithmetic, Common Difference = 6

$a_n = 10 + 6(n-1)$

$a_{25} = 10 + 6(24)$

$a_{25} = 154$

12) $-31,\ -33,\ -35,\ -37,\ \ldots$
Find a_{35}

Arithmetic, Common Difference = -2

$a_n = -31 + (-2)(n-1)$

$a_{35} = -31 + (-2)(34)$

$a_{35} = -99$

13) $1,\ 2,\ 6,\ 24,\ \ldots$
Find a_{20}

Not Arithmetic

Determine if the sequence is geometric. If it is, find the common ratio, the term named in the problem, and the explicit formula.

14) $1,\ 4,\ 16,\ 64,\ \ldots$
Find a_9

Geometric, Common Ratio = 4

$a_n = (1)(4)^{n-1}$ OR $4^{(n-1)}$

$a_9 = 65536$

15) $-7,\ -5,\ -2,\ 2,\ \ldots$
Find a_{10}

Not Geometric

16) $1,\ -2,\ 4,\ -8,\ \ldots$
Find a_{10}

Geometric, Common Ratio = -2

$a_n = (1)(-2)^{n-1}$

$a_{10} = -512$

14.2 Arithmetic and Geometric Sequences

$$S_n = \frac{n(a_1 + a_n)}{2}$$ sum

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$ sum

For numbers 16 – 20, find the sum of the first ***n*** terms indicated in part (a). Then, for part (b), find ***n*** for the given sum S_n.

17. $1 + 4 + 16 + 64 + \ldots$ ⇒ GEOMETRIC ($r = 4$)

a. $n = 14$
b. $S_n = 341$

a. $S_n = 1\left(\frac{1-4^n}{1-4}\right) = 89,478,495$

b. $341 = 1\left(\frac{1-4^n}{1-4}\right)$
$341 = \frac{1-4^n}{-3}$
$1023 = 1 - 4^n$
$1024 = -4^n$
$\log 1022 = n \log 4$
$\frac{\log 1022}{\log 4} = -n$
$5 = n$

18. $50 + 42 + 34 + 26 + \ldots$ ⇒ ARITHMETIC

a. $n = 40$
b. $S_n = 182$

$a_n = 50 + (-8)(39) = -262$
$S_{40} = \frac{40(50 + (-262))}{2} = -4240$
$182 = \frac{n(50 + (50 - 8(n-1)))}{2}$
$364 = n(50 + 50 - 8n + 8)$
$364 = -8n^2 + 108n$
$0 = -8n^2 + 108n - 364$
$0 = 2n^2 - 27n + 91$
$0 = (2n - 13)(n - 7)$
$n = \frac{13}{2}$ or 7 (no fractions)
$n = 7$

19. $7 + (-21) + 63 + (-189) + \ldots$ ⇒ GEOMETRIC

a. $n = 18$
b. $S_n = 3829$

$S_{18} = 7\left(\frac{1-(-3)^{18}}{1-(-3)}\right) = -677,985,854$
$3829 = 7\left(\frac{1-(-3)^n}{1-(-3)}\right)$
$547 = \frac{1-(-3)^n}{4}$
$2188 = 1 - (-3)^n$
$\log 2187 = n \log(-(-3))$
$\log 2187 = n \log 3$
$\frac{\log 2187}{\log 3} = n$
$7 = n$

b. $n = 7$

20. $2 + 16 + 30 + 44 + 58 + \ldots$ ⇒ ARITHMETIC

a. $n = 24$
b. $S_n = 2178$

$S_{24} = \frac{24(2 + 1(24-1)2)}{2} = 3912$
$2178 = \frac{n(2 + 2 + 14(n-1))}{2}$
$4356 = 14n^2 - 10n$
$0 = 14n^2 - 10n - 4356$
$0 = 2(7n^2 - 5n - 2178)$
Quad Formula $n = 18$ (exclude decimal ans)

21. $1 + 9 + 81 + 729 + \ldots$ GEOMETRIC

a. $n = 10$
b. $S_n = 820$

a. $S_{10} = 1\left(\frac{1-9^{10}}{1-9}\right) = 4358,48050$
b. $820 = 1\left(\frac{1-9^n}{1-9}\right)$
$-6560 = 1 - 9^n$
$-6561 = -9^n$
$6561 = 9^n$
$n = \log_9 6561 = 4$

22. $3 + 8 + 13 + 18 + 23 + \ldots$ ⇒ ARITHMETIC

a. $n = 20$
b. $S_n = 366$

a. $S_{20} = \frac{20(3 + 3 + 5(19))}{2} = 1010$
b. $366 = \frac{n(3 + 3 + 5(n-1))}{2}$
$732 = n(5n - 1)$
$0 = 5n^2 + n - 732$
Quad Formula. $n = 12$ or $n = -\frac{122}{5}$ exclude negative

Evaluate each series.

23. $\sum_{i=2}^{7} i + 2$

$(2+2) + (3+2) + (4+2) + (5+2) + (6+2) + (7+2)$
$= 2(2 + 3 + 4 + 5 + 6 + 7) = 39$

24. $\sum_{j=1}^{3} j^j$

$1^1 + 2^2 + 3^3$
$= 32$

25. $\sum_{k=3}^{5} t^k$

$= t^3 + t^4 + t^5$

Write each series in sigma notation.

26. $16 + 25 + 36 + 49 + 64$

$\sum_{k=4}^{8} k^2$

27. $2 + 4 + 8 + 16 + 32$

$\sum_{n=1}^{5} 2^n$

28. $501 + 502 + 503 + 504$

$\sum_{k=1}^{4} k + 500$
OR $\sum_{k=501}^{504} k$

Skills Review! Write the equation of a line with the given slope that passes through the given point.

In slope-intercept form: $y = mx + b$		In point-slope form: $y - y_1 = m(x - x_1)$	
1. slope = -3; through (-1,3)	2. slope = 0; through (-2,3)	3. slope = 3; through (1,-3)	4. slope = $-\frac{3}{5}$; through (0,0)