Pre-Calculus

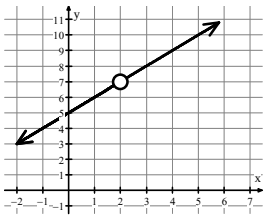
Write your questions and thoughts here!

15.1 Limits (Analytically)

Name:_____

Recall: Find the limit graphically.

$$\lim_{x\to 2}\frac{x^2+3x-10}{x-2}$$



A limit is…

Steps to Finding Limits Analytically

1.

2.

a.

b.

Direct Substitution:

1. $\lim_{x\to -1}(x^2+2x-4)$

2. $\lim_{x\to 2}\sqrt{3x-2}$

3. $\lim_{x\to 4}5$

Factor and Cancel:

4. $\lim_{x\to 0}\frac{4x^2-5x}{x}$

5. $\lim_{x\to -7}\frac{2x^2+13x-7}{x+7}$

6. $\lim_{x\to -2}\frac{3x^3-6x^2-24x}{3x^2+6x}$

Rationalize:

7. $\lim_{x\to 0}\frac{\sqrt{x+5}-\sqrt{5}}{x}$

8. $\lim_{x\to 5}\frac{\sqrt{x+4}-3}{x-5}$

15.1 Limits (Analytically)

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Two variables:

9. $\lim_{h\to 0}\dfrac{(x+h)^2-3(x+h)-(x^2-3x)}{h}$

10. $\lim_{h\to 0}\dfrac{2(x+h)^2+5(x+h)-1-(2x^2+5x-1)}{h}$

Now summarize what you learned!

15.1 Practice – Limits Analytically

Pre-Calculus

Find the value of each limit. No calculator is allowed.

1. $\lim_{x\to -2}(3x^2-x+1)$	2. $\lim_{x\to 1} 3$	3. $\lim_{x\to 5}\sqrt{4x-9}$	4. $\lim_{x\to \pi}\cos x$
5. $\lim_{x\to 0}\dfrac{x^2+2x-8}{x-4}$	6. $\lim_{x\to 5}(x+1)^2$	7. $\lim_{x\to 1}\dfrac{x^2-5x}{x-1}$	8. $\lim_{x\to -2}\dfrac{x^2-4x-10}{x}$

9. $\lim_{x\to -7} \dfrac{2x^3+11x^2-21x}{x^2+7x}$

10. $\lim_{x\to 0} \dfrac{\sqrt{x+7}-\sqrt{7}}{x}$

11. $\lim_{x\to 1} \dfrac{\sqrt{x+5}+\sqrt{6}}{x}$

12. $\lim_{x\to \frac{\pi}{8}} \sin(4x)$

13. $\lim_{x\to -1} \sqrt{3-x}$

14. $\lim_{x\to 2} \dfrac{\sqrt{5x-6}}{x}$

15. $\lim_{x\to 2} (x-x^2)$

16. $\lim_{x\to 0} (-2)$

17. $\lim_{x\to 8} \dfrac{x^2+2x-80}{x-8}$

18. $\lim_{x\to 4} \dfrac{5x^2-21x+4}{x-4}$

19. $\lim_{x\to 1} \dfrac{x^2+x-30}{x-1}$

20. $\lim_{x\to 0} \dfrac{3x^2+5x}{x}$

21. $\lim_{x\to -3} 14$

22. $\lim_{x\to \frac{\pi}{2}} \tan\left(\dfrac{x}{2}\right)$

23. $\lim_{x\to \frac{1}{2}} \dfrac{1-x-2x^2}{2x-1}$

24. $\lim_{h\to 0} \dfrac{5\sqrt{x+h}-5\sqrt{x}}{h}$

25.
$$\lim_{h\to 0}\frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$$

26.
$$\lim_{x\to 7}\frac{\sqrt{x+9}-4}{x-7}$$

27.
$$\lim_{x\to 0}\frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}$$

28.
$$\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$$

29.
$$\lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}$$

30.
$$\lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

31.
$$\lim_{x\to 5}\frac{2x^2-17x+35}{5-x}$$

32.
$$\lim_{x\to 3}(2x^2+5x-6)$$

33.
$$\lim_{h\to 0}\frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$$

34.
$$\lim_{x\to 0}\frac{\sqrt{x+11}-\sqrt{11}}{x}$$

35.
$$\lim_{x\to\frac{1}{3}}\frac{6x^2+13x-5}{3x-1}$$

36.
$$\lim_{h\to 0}\frac{6-3(x+h)-(6-3x)}{h}$$

37.
$$\lim_{x\to 2}\frac{x^2+6x-16}{2-x}$$

15.1 Extension

One-sided limits. If we have $\lim_{x\to 3^-} f(x)$, this means we approach the x-value of 3 from the negative (left) side. In other words, you start on the left of 3 and move to the right towards 3. For a limit to exist, the left AND right side limits must be the same.

Using the following piecewise functions, find the given values.

$$g(x)=\begin{cases}\sqrt{5-x}, & x<-4\\ x^2-5, & -4\le x<2\\ x-3, & x\ge 2\end{cases}$$

$\lim_{x\to 2^-} g(x)=$

$\lim_{x\to -4^+} g(x)=$

$g(2)=$

$\lim_{x\to -4^-} g(x)=$

$\lim_{x\to 2^+} g(x)=$

$\lim_{x\to 2} g(x)=$

$\lim_{x\to -4} g(x)=$

$g(-4)=$

$$h(x)=\begin{cases}-|x|, & x\le -5\\ 20-x^2, & -5<x\le 3\\ 4x-1, & x>3\end{cases}$$

$\lim_{x\to -5^+} h(x)=$

$\lim_{x\to -5} h(x)=$

$h(3)=$

$\lim_{x\to -5^-} h(x)=$

$\lim_{x\to 3^+} h(x)=$

$\lim_{x\to 3} h(x)=$

$h(-5)=$

$\lim_{x\to 3^-} h(x)=$

$$w(\theta)=\begin{cases}\sin\theta, & \theta\le\pi\\ \cos\theta, & \pi<\theta<2\pi\\ \tan\theta, & \theta>2\pi\end{cases}$$

$\lim_{x\to \pi^-} w(\theta)=$

$w(\pi)=$

$\lim_{x\to \pi^+} w(\theta)=$

$\lim_{x\to 2\pi^-} w(\theta)=$

$\lim_{x\to \pi} w(\theta)=$

$\lim_{x\to 2\pi^+} w(\theta)=$

$\lim_{x\to 2\pi} w(\theta)=$

$w(2\pi)=$

Skillz Review: Using the graph, find each value.

a. $\lim_{x\to 1^-} f(x)=$

b. $f(-1)=$

c. $\lim_{x\to -1} f(x)=$

d. $\lim_{x\to -2} f(x)=$

e. $f(1)=$

f. $\lim_{x\to 1^+} f(x)=$