15.1 Practice – Limits Analytically

Pre-Calculus

Solutions

Find the value of each limit. No calculator is allowed.

1. $\lim_{x\to -2}(3x^2 - x + 1)$

$3(-2)^2 - (-2) + 1$

$\boxed{15}$

2. $\lim_{x\to 1} 3$

$\boxed{3}$

3. $\lim_{x\to 5}\sqrt{4x-9}$

$\sqrt{4(5)-9}$

$\boxed{\sqrt{11}}$

4. $\lim_{x\to \pi}\cos x$

$\cos(\pi)$

$\boxed{-1}$

5. $\lim_{x\to 0}\dfrac{x^2+2x-8}{x-4}$

$\dfrac{(0)^2+2(0)-8}{(0)-4}$

$\boxed{2}$

6. $\lim_{x\to 5}(x+1)^2$

$(5+1)^2$

$\boxed{36}$

7. $\lim_{x\to 1}\dfrac{x^2-5x}{x-1}$

$\lim_{x\to 1}\dfrac{x(x-5)}{x-1}$

$\boxed{\text{Does not exist.}}$

8. $\lim_{x\to -2}\dfrac{x^2-4x-10}{x}$

$\dfrac{(-2)^2-4(-2)-10}{(-2)}$

$\boxed{-1}$

9. $\lim_{x\to -7}\dfrac{2x^3+11x^2-21x}{x^2+7x}$

$\lim_{x\to -7}\dfrac{\cancel{x}\cancel{(x+7)}(2x-3)}{\cancel{x}\cancel{(x+7)}}$

$2(-7)-3$

$\boxed{-17}$

10. $\lim_{x\to 0}\dfrac{\sqrt{x+7}-\sqrt{7}}{x}\cdot\dfrac{\sqrt{x+7}+\sqrt{7}}{\sqrt{x+7}+\sqrt{7}}$

$\lim_{x\to 0}\dfrac{(x+7)-(7)}{x(\sqrt{x+7}+\sqrt{7})}$

$\lim_{x\to 0}\dfrac{\cancel{x}}{\cancel{x}(\sqrt{x+7}+\sqrt{7})}$

$\dfrac{1}{\sqrt{0+7}+\sqrt{7}} = \boxed{\dfrac{1}{2\sqrt{7}}}$

11. $\lim_{x\to 1}\dfrac{\sqrt{x+5}+\sqrt{6}}{x}$

$\dfrac{\sqrt{(1)+5}+\sqrt{6}}{(1)}$

$\boxed{2\sqrt{6}}$

12. $\lim_{x\to \frac{\pi}{8}}\sin(4x)$

$\sin\left(4\left(\frac{\pi}{8}\right)\right)$

$\sin\left(\frac{\pi}{2}\right)$

$\boxed{1}$

13. $\lim_{x\to -1}\sqrt{3-x}$

$\sqrt{3-(-1)}$

$\boxed{2}$

14. $\lim_{x\to 2}\dfrac{\sqrt{5x-6}}{x}$

$\dfrac{\sqrt{5(2)-6}}{(2)}$

$\boxed{1}$

15. $\lim_{x\to 2}(x-x^2)$

$(2)-(2)^2$

$\boxed{-2}$

16. $\lim_{x\to 0}(-2)$

$\boxed{-2}$

17. $\lim_{x\to 8}\dfrac{x^2+2x-80}{x-8}$

$\lim_{x\to 8}\dfrac{(x+10)\cancel{(x-8)}}{\cancel{x-8}}$

$(8)+10 = \boxed{18}$

18. $\lim_{x\to 4}\dfrac{5x^2-21x+4}{x-4}$

$\lim_{x\to 4}\dfrac{\cancel{(x-4)}(5x-1)}{\cancel{(x-4)}}$

$5(4)-1 = \boxed{19}$

19. $\lim_{x\to 1}\dfrac{x^2+x-30}{x-1}$

$\lim_{x\to 1}\dfrac{(x-5)(x+6)}{x-1}$

$\boxed{\text{Does not exist}}$

20. $\lim_{x\to 0}\dfrac{3x^2+5x}{x}$

$\lim_{x\to 0}\dfrac{\cancel{x}(3x+5)}{\cancel{x}}$

$3(0)+5 = \boxed{5}$

21. $\lim_{x\to -3} 14$

$\boxed{14}$

22. $\lim_{x\to \frac{\pi}{2}}\tan\left(\frac{x}{2}\right)$

$\tan\left(\frac{\frac{\pi}{2}}{2}\right)$

$\tan\left(\frac{\pi}{4}\right) = \boxed{1}$

23. $\lim_{x\to \frac{1}{2}}\dfrac{1-x-2x^2}{2x-1}$

$\lim_{x\to \frac{1}{2}} -\dfrac{\cancel{(2x-1)}(x+1)}{\cancel{(2x-1)}}$

$-\left(\left(\frac{1}{2}\right)+1\right) = \boxed{-\frac{3}{2}}$

24. $\lim_{h\to 0}\dfrac{5\sqrt{x+h}-5\sqrt{x}}{h}\cdot\dfrac{5\sqrt{x+h}+5\sqrt{x}}{5\sqrt{x+h}+5\sqrt{x}}$

$\lim_{h\to 0}\dfrac{25(x+h)-25x}{h(5\sqrt{x+h}+5\sqrt{x})}$

$\lim_{h\to 0}\dfrac{25\cancel{h}}{\cancel{h}(5\sqrt{x+h}+5\sqrt{x})}$

$\dfrac{25}{5\sqrt{x}+5\sqrt{x}} = \dfrac{25}{10\sqrt{x}} = \boxed{\dfrac{5}{2\sqrt{x}}}$

25. $\lim_{h\to0}\frac{(x+h)^2+6(x+h)-(x^2+6x)}{h}$

$\lim_{h\to0}\frac{x^2+2hx+h^2+6x+6h-x^2-6x}{h}$

$\lim_{h\to0}\frac{2hx+h^2+6h}{h}$

$\lim_{h\to0}\frac{\cancel{h}(2x+h+6)}{\cancel{h}} = \boxed{2x+6}$

26. $\lim_{x\to7}\frac{\sqrt{x+9}-4}{x-7}\cdot\frac{\sqrt{x+9}+4}{\sqrt{x+9}+4}$

$\lim_{x\to7}\frac{x+9-16}{(x-7)(\sqrt{x+9}+4)}$

$\lim_{x\to7}\frac{\cancel{x-7}}{\cancel{(x-7)}(\sqrt{x+9}+4)}$

$\frac{1}{\sqrt{7+9}+4} = \boxed{\frac{1}{8}}$

27. $\lim_{x\to0}\frac{\frac{1}{(x+2)^2}-\frac{1}{4}}{x}\cdot\frac{4(x+2)^2}{4(x+2)^2}$

$\lim_{x\to0}\frac{4-(x+2)^2}{4x(x+2)^2}$

$\lim_{x\to0}\frac{4-(x^2+4x+4)}{4x(x+2)^2}$

$\lim_{x\to0}\frac{4-x^2-4x-4}{4x(x+2)^2}$

$\lim_{x\to0}\frac{-\cancel{x}(x+4)}{4\cancel{x}(x+2)^2}$

$\frac{-4}{4(2)^2} = \boxed{-\frac{1}{4}}$

28. $\lim_{x\to0}\frac{\sqrt{x+9}-3}{x}\cdot\frac{\sqrt{x+9}+3}{\sqrt{x+9}+3}$

$\lim_{x\to0}\frac{x+9-9}{x(\sqrt{x+9}+3)}$

$\lim_{x\to0}\frac{1}{\sqrt{x+9}+3}$

$\boxed{\frac{1}{6}}$

29. $\lim_{x\to0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x}\cdot\frac{3(x+3)}{3(x+3)}$

$\lim_{x\to0}\frac{3-(x+3)}{3x(x+3)}$

$\lim_{x\to0}\frac{-1}{3(x+3)} = \boxed{-\frac{1}{9}}$

30. $\lim_{h\to0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h}\cdot\frac{x(x+h)}{x(x+h)}$

$\lim_{h\to0}\frac{x-(x+h)}{xh(x+h)}$

$\lim_{h\to0}\frac{-h}{xh(x+h)}$

$\frac{-1}{x(x+0)} = \boxed{-\frac{1}{x^2}}$

31. $\lim_{x\to5}\frac{2x^2-17x+35}{5-x}$

$\lim_{x\to5}\frac{(x-5)(2x-7)}{-(x-5)}$

$-(2(5)-7)$

$\boxed{-3}$

32. $\lim_{x\to3}(2x^2+5x-6)$

$2(3)^2+5(3)-6$

$\boxed{27}$

33. $\lim_{h\to0}\frac{4(x+h)^2-5(x+h)-2-(4x^2-5x-2)}{h}$

$\lim_{h\to0}\frac{4(x^2+2hx+h^2)-5x-5h-2-4x^2+5x+2}{h}$

$\lim_{h\to0}\frac{4x^2+8hx+4h^2-5x-5h-2-4x^2+5x+2}{h}$

$\lim_{h\to0}\frac{8hx+4h^2-5h}{h} = \lim_{h\to0}\frac{h(8x+4h-5)}{h} = \boxed{8x-5}$

34. $\lim_{x\to0}\frac{\sqrt{x+11}-\sqrt{11}}{x}\cdot\frac{\sqrt{x+11}+\sqrt{11}}{\sqrt{x+11}+\sqrt{11}}$

$\lim_{x\to0}\frac{x+11-11}{x(\sqrt{x+11}+\sqrt{11})}$

$\lim_{x\to0}\frac{x}{x(\sqrt{x+11}+\sqrt{11})}$

$\boxed{\frac{1}{2\sqrt{11}}}$

35. $\lim_{x\to\frac{1}{3}}\frac{6x^2+13x-5}{3x-1}$

$\lim_{x\to\frac{1}{3}}\frac{\cancel{(3x-1)}(2x+5)}{\cancel{(3x-1)}}$

$2(\frac{1}{3})+5 = \frac{2}{3}+\frac{15}{3} = \boxed{\frac{17}{3}}$

36. $\lim_{h\to0}\frac{6-3(x+h)-(6-3x)}{h}$

$\lim_{h\to0}\frac{6-3x-3h-6+3x}{h}$

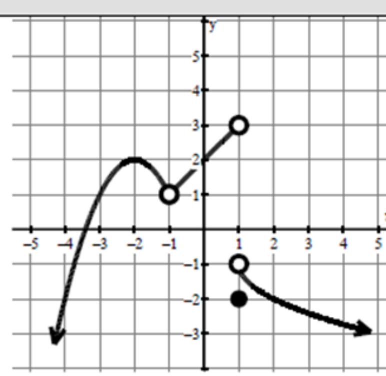
$\lim_{h\to0}\frac{-3h}{h} = \boxed{-3}$

37. $\lim_{x\to2}\frac{x^2+6x-16}{2-x}$

$\lim_{x\to2}\frac{(x-2)(x+8)}{-(x-2)}$

$-(2+8) = \boxed{-10}$

Skillz Review: Using the graph, find each value.



a. $\lim_{x\to1^-} f(x) = 3$

b. $f(-1) =$ DNE

c. $\lim_{x\to-1} f(x) = 1$

d. $\lim_{x\to-2} f(x) = 2$

e. $f(1) = -2$

f. $\lim_{x\to1^+} f(x) = -1$