**15.2 Practice – Definition of the Derivative**

Pre-Calculus

Name: Solutions

**Find the derivative using limits. If the equation is given as $y =$, use Leibniz Notation: $\frac{dy}{dx}$. If the equation is given as $f(x) =$, use Lagrange Notation: $f'(x)$. WRITE SMALL!!**

1. $y = 5x + 1$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{5(x+h)+1-(5x+1)}{h}$$
$$= \lim_{h\to 0} \frac{5x+5h+1-5x-1}{h}$$
$$= \lim_{h\to 0} \frac{5h}{h}$$
$$\boxed{\frac{dy}{dx} = 5}$$

2. $f(x) = 7 - 6x$

$$f'(x) = \lim_{h\to 0} \frac{7-6(x+h)-(7-6x)}{h}$$
$$= \lim_{h\to 0} \frac{7-6x-6h-7+6x}{h}$$
$$= \lim_{h\to 0} \frac{-6h}{h}$$
$$\boxed{f'(x) = -6}$$

3. $y = 31$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{31-31}{h}$$
$$= \lim_{h\to 0} \frac{0}{h}$$
$$\boxed{\frac{dy}{dx} = 0}$$

4. $y = 5x^2 - x$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{5(x+h)^2-(x+h)-(5x^2-x)}{h}$$
$$\lim_{h\to 0} \frac{5(x^2+2hx+h^2)-x-h-5x^2+x}{h}$$
$$\lim_{h\to 0} \frac{5x^2+10hx+5h^2-h-5x^2}{h}$$
$$\lim_{h\to 0} \frac{10hx+5h^2-h}{h}$$
$$\lim_{h\to 0} \frac{h(10x+5h-1)}{h}$$
$$\boxed{\frac{dy}{dx} = 10x-1}$$

5. $f(x) = 4$

$$f'(x) = \lim_{h\to 0} \frac{4-4}{h}$$
$$= \lim_{h\to 0} \frac{0}{h}$$
$$\boxed{f'(x) = 0}$$

6. $f(x) = 2 + 10x - x^2$

$$f'(x) = \lim_{h\to 0} \frac{2+10(x+h)-(x+h)^2-(2+10x-x^2)}{h}$$
$$= \lim_{h\to 0} \frac{2+10x+10h-(x^2+2hx+h^2)-2-10x+x^2}{h}$$
$$= \lim_{h\to 0} \frac{10h-x^2-2hx-h^2+x^2}{h}$$
$$\lim_{h\to 0} \frac{10h-2hx-h^2}{h}$$
$$\lim_{h\to 0} \frac{h(10-2x-h)}{h}$$
$$\boxed{f'(x) = 10-2x}$$

7. $y = 3x^2 - 2x + 8$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{3(x+h)^2 - 2(x+h) + 8 - (3x^2 - 2x + 8)}{h}$$

$$\lim_{h\to 0} \frac{3(x^2+2hx+h^2) - 2x - 2h - 3x^2 + 2x - 8 + 8}{h}$$

$$\lim_{h\to 0} \frac{3x^2 + 6hx + 3h^2 - 2h - 3x^2}{h}$$

$$\lim_{h\to 0} \frac{h(6x+3h-2)}{h}$$

$$\boxed{\frac{dy}{dx} = 6x - 2}$$

8. $f(x) = \sqrt{2x-1}$

$$\lim_{h\to 0} \frac{\sqrt{2(x+h)-1} - \sqrt{2x-1}}{h} \cdot \frac{\sqrt{2(x+h)-1} + \sqrt{2x-1}}{\sqrt{2(x+h)-1} + \sqrt{2x-1}}$$

$$\lim_{h\to 0} \frac{(2x+2h-1) - (2x-1)}{h(\sqrt{2x+2h-1} + \sqrt{2x-1})}$$

$$\lim_{h\to 0} \frac{2h}{h(\sqrt{2x+2h-1} + \sqrt{2x-1})}$$

$$f'(x) = \frac{2}{\sqrt{2x-1} + \sqrt{2x-1}}$$

$$f'(x) = \frac{2}{2\sqrt{2x-1}} = \boxed{\frac{1}{\sqrt{2x-1}}}$$

9. $y = \sqrt{5x+2}$

$$\lim_{h\to 0} \frac{\sqrt{5(x+h)+2} - \sqrt{5x+2}}{h} \cdot \frac{\sqrt{5(x+h)+2} + \sqrt{5x+2}}{\sqrt{5(x+h)+2} + \sqrt{5x+2}}$$

$$\lim_{h\to 0} \frac{(5x+5h+2) - (5x+2)}{h(\sqrt{5x+5h+2} + \sqrt{5x+2})}$$

$$\lim_{h\to 0} \frac{5h}{h(\sqrt{5x+5h+2} + \sqrt{5x+2})}$$

$$f'(x) = \frac{5}{\sqrt{5x+2} + \sqrt{5x+2}}$$

$$\boxed{f'(x) = \frac{5}{2\sqrt{5x+2}}}$$

10. $f(x) = 2 - \sqrt{6x+5}$

$$\lim_{h\to 0} \frac{2 - \sqrt{6(x+h)+5} - (2 - \sqrt{6x+5})}{h}$$

$$\lim_{h\to 0} \frac{-\sqrt{6x+6h+5} + \sqrt{6x+5}}{h} \cdot \frac{(-\sqrt{6x+6h+5} - \sqrt{6x+5})}{(-\sqrt{6x+6h+5} - \sqrt{6x+5})}$$

$$\lim_{h\to 0} \frac{(6x+6h+5) - (6x+5)}{h(-\sqrt{6x+6h+5} - \sqrt{6x+5})}$$

$$\lim_{h\to 0} \frac{6h}{h(-\sqrt{6x+6h+5} - \sqrt{6x+5})}$$

$$f'(x) = \frac{6}{-2\sqrt{6x+5}} = \boxed{-\frac{3}{\sqrt{6x+5}}}$$

11. $f(x) = \frac{1}{3x-2}$

$$\lim_{h\to 0} \frac{\frac{1}{3(x+h)-2} - \frac{1}{3x-2}}{h} \cdot \frac{[3(x+h)-2](3x-2)}{[3(x+h)-2](3x-2)}$$

$$\lim_{h\to 0} \frac{3x-2-(3x+3h-2)}{h(3x+3h-2)(3x-2)}$$

$$\lim_{h\to 0} \frac{-3h}{h(3x+3h-2)(3x-2)}$$

$$f'(x) = \frac{-3}{(3x-2)(3x-2)} = \boxed{\frac{-3}{(3x-2)^2}}$$

12. $y = \frac{1}{5-x}$

$$\lim_{h\to 0} \frac{\frac{1}{5-(x+h)} - \frac{1}{5-x}}{h} \cdot \frac{(5-x-h)(5-x)}{(5-x-h)(5-x)}$$

$$\lim_{h\to 0} \frac{5-x-(5-x-h)}{h(5-x-h)(5-x)}$$

$$\lim_{h\to 0} \frac{h}{h(5-x-h)(5-x)}$$

$$\boxed{f'(x) = \frac{1}{(5-x)^2}}$$

**For each problem, create an equation of the tangent line of $f$ at the given point. The answer can be in point-slope form OR slope-intercept.**

13. $f(7) = 5$ and $f'(7) = -2$

$y - 5 = -2(x-7)$

or

$y = -2x + 19$

14. $f(-2) = 3$ and $f'(-2) = 4$

$y - 3 = 4(x+2)$

or

$y = 4x + 11$

15. $f(1) = -5$ and $f'(1) = 3$

$y + 5 = 3(x-1)$

or

$y = 3x - 8$

16. $f(x) = 3x^2 + 2x;\ f'(x) = 6x + 2;\ x = -2$

$3(-2)^2 + 2(-2)$

$3(4) - 4$

$f(-2) = 8$

$-12 + 2$

$f'(-2) = -10$

$\boxed{y - 8 = -10(x+2) \text{ or } y = -10x - 12}$

17. $f(x) = 10\sqrt{6x+1};\ f'(x) = \frac{30}{\sqrt{6x+1}};\ x = 4$

$10\sqrt{25}$

$f(4) = 50$

$\frac{30}{5}$

$f'(4) = 6$

$\boxed{y - 50 = 6(x-4) \text{ or } y = 6x + 26}$

18. $f(x) = \cos 2x;\ f'(x) = -2\sin 2x;\ x = \frac{\pi}{4}$

$\cos(2 \cdot \frac{\pi}{4})$

$\cos(\frac{\pi}{2})$

$f(\frac{\pi}{4}) = 0$

$-2\sin(2 \cdot \frac{\pi}{4})$

$-2\sin(\frac{\pi}{2})$

$f'(\frac{\pi}{4}) = -2$

$\boxed{y - 0 = -2(x - \frac{\pi}{4}) \text{ or } y = -2x + \frac{\pi}{2}}$

19. $f(x) = \tan x;\ f'(x) = \sec^2 x;\ x = \frac{\pi}{3}$

$\tan(\frac{\pi}{3})$

$f(\frac{\pi}{3}) = \sqrt{3}$

$\sec^2(\frac{\pi}{3})$

$\left[\frac{1}{\cos(\frac{\pi}{3})}\right]^2$

$f'(\frac{\pi}{3}) = \left(\frac{1}{1/2}\right)^2 = 4$

$\boxed{y - \sqrt{3} = 4(x - \frac{\pi}{3})}$ This form is the easiest. Trying to write it in slope-intercept form would be a bit crazy!

**Using the function listed, find the equation of the tangent line at the given x-value.**

20. $f(x) = 8x - 4;\ x = 2$

$f(2) = 16 - 4$

$f(2) = 12$

$$f'(x) = \lim_{h \to 0} \frac{8(x+h) - 4 - (8x - 4)}{h}$$

$$= \lim_{h \to 0} \frac{8x + 8h - 4 - 8x + 4}{h}$$

$$= \lim_{h \to 0} \frac{8h}{h}$$

$f'(x) = 8$ $\quad f'(2) = 8$

$\boxed{y - 12 = 8(x - 2) \text{ or } y = 8x - 4}$

21. $f(x) = 2x^2 - 5x + 1;\ x = -1$

$f(-1) = 2(-1)^2 + 5 + 1$

$f(-1) = 8$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h)^2 - 5(x+h) + 1 - (2x^2 - 5x + 1)}{h}$$

$$= \lim_{h \to 0} \frac{2(x^2 + 2hx + h^2) - 5x - 5h - 2x^2 + 5x}{h}$$

$$= \lim_{h \to 0} \frac{2x^2 + 4hx + 2h^2 - 5h - 2x^2}{h}$$

$$= \lim_{h \to 0} \frac{h(4x + 2h - 5)}{h}$$

$f'(x) = 4x - 5$ $\quad f'(-1) = -9$

$\boxed{y - 8 = -9(x+1) \text{ or } y = -9x - 1}$

**Skillz Review:** Using the graph, find each value.

a. $\lim_{x \to 3^-} f(x) =$ 4

b. $f(-1) =$ DNE

c. $\lim_{x \to -3} f(x) =$ 4

d. $\lim_{x \to -1} f(x) =$ 2

e. $f(-3) =$ 1

f. $\lim_{x \to 3^+} f(x) =$ −2

g. $f(3) =$ 4

h. $\lim_{x \to 0} f(x) =$ 3