15.3 Practice – Power Rule

Name: Solutions

Pre-Calculus

Find the derivative of each function and simplify.

1. $f(x) = 7x$

$f'(x) = 7$

2. $y = 37$

$\frac{dy}{dx} = 0$

3. $s(t) = -9t$

$s'(t) = -9$

4. $s(t) = 4$

$s'(t) = 0$

5. $y = x^2 - 8x + 10$

$\frac{dy}{dx} = 2x - 8$

6. $f(x) = \frac{x}{5}$

$f'(x) = \frac{1}{5}$

7. $f(x) = \pi^2$

$f'(x) = 0$

8. $f(x) = \frac{5}{x^2} = 5x^{-2}$

$f'(x) = -5x^{-3}$

$f'(x) = -\frac{5}{x^3}$

9. $y = 8\sqrt{x} = 8x^{\frac{1}{2}}$

$\frac{dy}{dx} = 4x^{-\frac{1}{2}} = \frac{4}{\sqrt{x}}$

10. $s(t) = \frac{3}{t^4} = 3t^{-4}$

$s'(t) = -12t^{-5}$

$= -\frac{12}{t^5}$

11. $y = \frac{2}{x^2} = 2x^{-2}$

$\frac{dy}{dx} = -4x^{-3}$

$= -\frac{4}{x^3}$

12. $f(x) = 9\sqrt{x} = 9x^{\frac{1}{2}}$

$f'(x) = \frac{9}{2}x^{-\frac{1}{2}}$

$= \frac{9}{2\sqrt{x}}$

13. $h(x) = 2e^3$

$h'(x) = 0$

14. $s(t) = \frac{6}{\sqrt{t}} = 6t^{-\frac{1}{2}}$

$s'(t) = -3t^{-\frac{3}{2}}$

$= -\frac{3}{\sqrt{t^3}}$

15. $y = \sqrt[3]{x} = x^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}}$

$= \frac{1}{3\sqrt[3]{x^2}}$

16. $f(x) = \sqrt[5]{x^7} = x^{\frac{7}{5}}$

$f'(x) = \frac{7}{5}x^{\frac{2}{5}}$

17. $y = 11x^5 - 3x + 13$

$y' = 55x^4 - 3$

18. $s(t) = 10 - 6t^3 + 7t$

$s'(t) = -18t^2 + 7$

19. $f(x) = e^6 + \pi^5 - 2$

$f'(x) = 0$

20. $f(x) = \frac{x}{x^{-5}} = x^6$

$f'(x) = 6x^5$

21. $y = (x^2 + 6x - 2)(2x^{-2} + x^{-4})$

$y = 2 + x^{-2} + 12x^{-1} + 6x^{-3} - 4x^{-2} - 2x^{-4}$

$y = 2 - 3x^{-2} + 12x^{-1} + 6x^{-3} - 2x^{-4}$

$\frac{dy}{dx} = 6x^{-3} - 12x^{-2} - 18x^{-4} + 8x^{-5}$

$\frac{8}{x^5} - \frac{18}{x^4} + \frac{6}{x^3} - \frac{12}{x^2}$

22. $h(x) = \sqrt{x}(\sqrt[3]{x} - \sqrt[4]{x})$

$= x^{\frac{1}{2}}(x^{\frac{1}{3}} - x^{\frac{1}{4}}) = x^{\frac{5}{6}} - x^{\frac{3}{4}}$

$h'(x) = \frac{5}{6}x^{-\frac{1}{6}} - \frac{3}{4}x^{-\frac{1}{4}}$

$h'(x) = \frac{5}{6\sqrt[6]{x}} - \frac{3}{4\sqrt[4]{x}}$

23. $h(x) = \frac{x^3 - 5x^2 + 7x}{x} = x^2 - 5x + 7$

$h'(x) = 2x - 5$

24. $y = \frac{3x^5 + 2x^2 - 4}{x^2} = 3x^3 + 2 - 4x^{-2}$

$y' = 9x^2 + 8x^{-3}$

$y' = 9x^2 + \frac{8}{x^3}$

Find the value of the derivative of the function at the indicated point.

25. $f(x) = \frac{1}{x^2}$ at $(1,1)$

$f'(x) = -\frac{2}{x^3}$

$f'(1) = -2$

26. $f(x) = 8 - \frac{2}{3x}$ at $\left(\frac{2}{3}, 7\right)$

$f'(x) = \frac{2}{3x^2}$

$f'(\frac{2}{3}) = \frac{2}{3(\frac{4}{9})} = \frac{2}{\frac{4}{3}}$

$2 \cdot \frac{3}{4} = \frac{3}{2}$

27. $f(x) = \frac{1}{3\sqrt{x}}$ at $\left(4, \frac{1}{6}\right)$

$f(x) = \frac{1}{3}x^{-\frac{1}{2}}$

$f'(x) = -\frac{1}{6}x^{-\frac{3}{2}} = -\frac{1}{6\sqrt{x^3}}$

$f'(4) = -\frac{1}{6\sqrt{4^3}} = -\frac{1}{6(8)} = -\frac{1}{48}$

Determine the x-value(s) at which the function has a horizontal tangent line.

28. $y = x^4 - 8x^2 + 2$

$4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$4x(x-2)(x+2) = 0$

$x = 0, 2, -2$

29. $y = x^3 - x$

$3x^2 - 1 = 0$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} = \pm\frac{\sqrt{3}}{3}$

30. $y = x^2 + 1$

$2x = 0$

$x = 0$

31. $y = \frac{1}{x^2}$

$-\frac{2}{x^3} = 0$

$-2 = 0$?

There is no point on this graph where a horizontal tangent exists.

Find the equation of a tangent line of each function at the indicated point.

32. $f(x) = 2x^2 + 7x + 3; \quad x = -1$

$f(-1) = 2 - 7 + 3$

$f(-1) = -2$

$f'(x) = 4x + 7$

$f'(-1) = 3$

$y + 2 = 3(x + 1)$

33. $f(x) = 2x^3 - 5x; \quad x = -2$

$f(-2) = -16 + 10$

$f(-2) = -6$

$f'(x) = 6x^2 - 5$

$f'(-2) = 19$

$y + 6 = 19(x + 2)$

34. $f(x) = \frac{16}{x} - \frac{x}{2}; \quad x = 4$

$f(4) = 4 - 2$

$f(4) = 2$

$f'(x) = -\frac{16}{x^2} - \frac{1}{2}$

$f'(4) = -\frac{3}{2}$

$y - 2 = -\frac{3}{2}(x - 4)$

35. $f(x) = \frac{4}{\sqrt{x}} - x; \quad x = 9$

$f(9) = \frac{4}{3} - 9$

$f(9) = -\frac{23}{3}$

$f'(x) = -\frac{2}{\sqrt{x^3}} - 1$

$f'(9) = -\frac{2}{27} - \frac{27}{27}$

$f'(9) = -\frac{29}{27}$

$y + \frac{23}{3} = -\frac{29}{27}(x - 9)$

Skills Review: Using the graph, find each value.

a. $\lim_{x \to 2^-} f(x) = 3$

b. $f(3) = 1$

c. $\lim_{x \to 0} f(x) = 1$

d. $\lim_{x \to -2^+} f(x) = -1$

e. $f(-2) =$ DNE

f. $\lim_{x \to -2^-} f(x) = -2$

g. $\lim_{x \to -2} f(x) =$ DNE

h. $f(2) = 3$