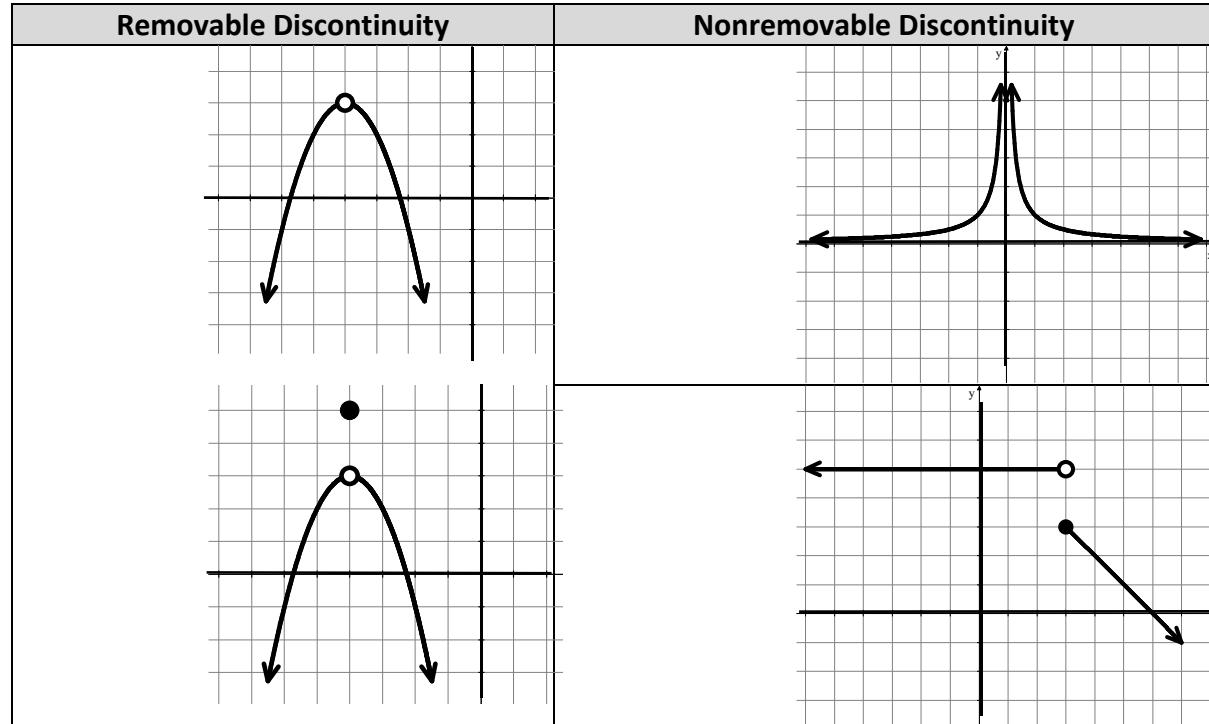
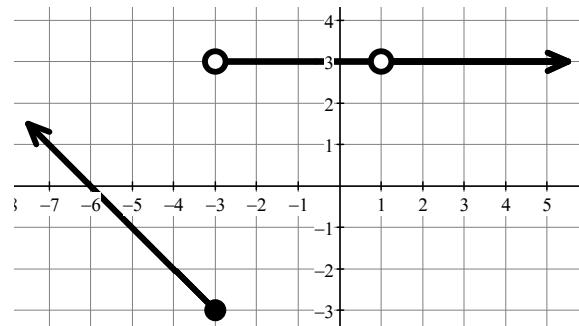


**2.3 Limits Graphically****Continuous Functions and Discontinuities**

A **continuous function** is a function that \_\_\_\_\_

**Classifying Discontinuities**

- Using the graph at the right, identify the  $x$ -values of each discontinuity, and write if it is removable or not. If it is nonremovable then classify what type.

**Limits**

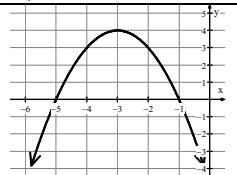
A **one-sided limit** is the \_\_\_\_\_ that a function approaches from either the \_\_\_\_\_ or the \_\_\_\_\_ side of a given \_\_\_\_\_.

A **limit** is the \_\_\_\_\_ that a function \_\_\_\_\_ at a given \_\_\_\_\_ if the left-sided limit is equal to the right-sided limit.

Write your questions and thoughts here!

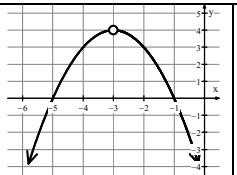
## 2.3 Limits Graphically

2)



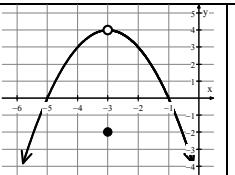
$$\lim_{x \rightarrow -3} f(x) =$$

$$f(-3) =$$



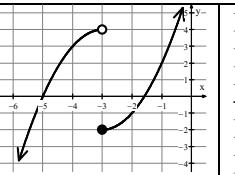
$$\lim_{x \rightarrow -3} f(x) =$$

$$f(-3) =$$



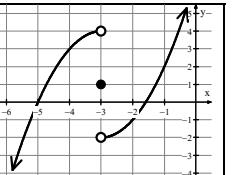
$$\lim_{x \rightarrow -3} f(x) =$$

$$f(-3) =$$



$$\lim_{x \rightarrow -3} f(x) =$$

$$f(-3) =$$



$$\lim_{x \rightarrow -3} f(x) =$$

$$f(-3) =$$

3)

a)  $\lim_{x \rightarrow -2^-} f(x) =$

b)  $\lim_{x \rightarrow -2^+} f(x) =$

c)  $\lim_{x \rightarrow -2} f(x) =$

d)  $\lim_{x \rightarrow 1} f(x) =$

e)  $\lim_{x \rightarrow 0} f(x) =$

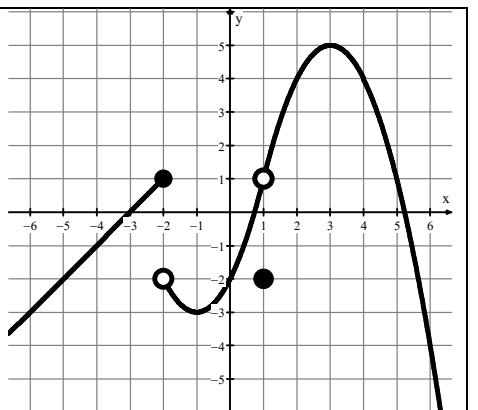
f)  $\lim_{x \rightarrow 3^-} f(x) =$

g)  $\lim_{x \rightarrow -1} f(x) =$

h)  $\lim_{x \rightarrow -3} f(x) =$

i)  $f(-2) =$

j)  $f(1) =$



4)



Write T (true) or F (false) under each statement. Use the graph on the right.

a)  $\lim_{x \rightarrow -1^+} f(x) = 1$

b)  $\lim_{x \rightarrow 2} f(x) = 2$

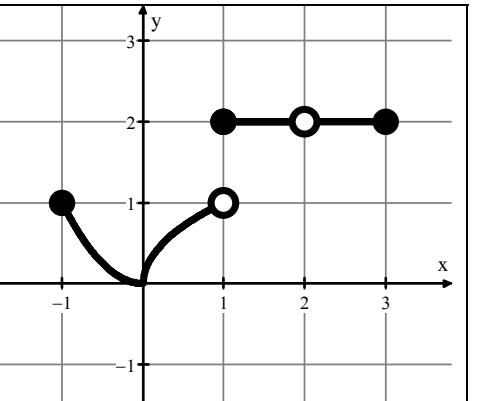
c)  $\lim_{x \rightarrow 1^-} f(x) = 1$

d)  $\lim_{x \rightarrow 1^+} f(x) = 2$

e)  $\lim_{x \rightarrow 1} f(x) =$  does not exist

f)  $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x)$

g)  $\lim_{x \rightarrow 2} f(x) =$  does not exist



### Continuous Function – formal definition

For  $f(x)$  to be continuous, the following three conditions must be met:

1.

2.

3.

Now summarize what you learned!

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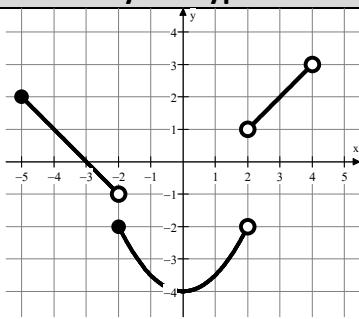
## 2.3 Practice – Limits (Graphically)

Name: \_\_\_\_\_

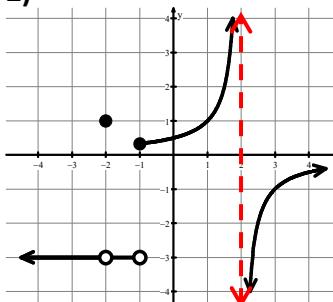
Pre-Calculus

**For 1-4, identify the  $x$ -values of each discontinuity, and write if it is removable or not. If it is nonremovable then classify the type.**

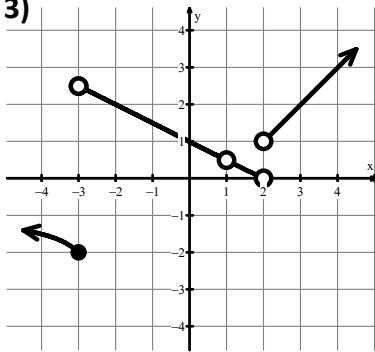
1)



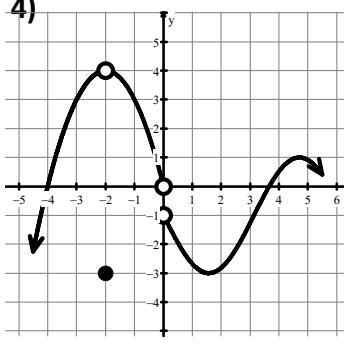
2)



3)



4)



**For 5-8, give the value of each statement. If the value does not exist, write "does not exist" or "undefined."**

5)

a.  $\lim_{x \rightarrow -1^-} f(x) =$

b.  $f(1) =$

c.  $\lim_{x \rightarrow 0} f(x) =$

d.  $\lim_{x \rightarrow 2^+} f(x) =$

e.  $f(-1) =$

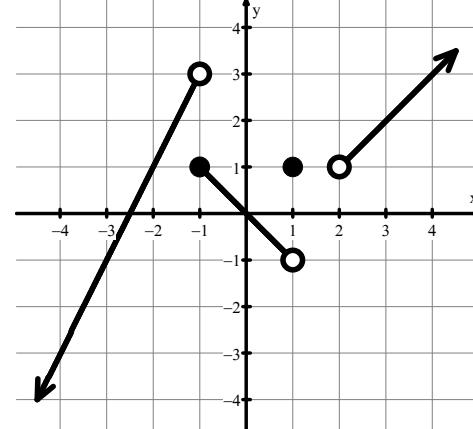
f.  $\lim_{x \rightarrow 1^-} f(x) =$

g.  $\lim_{x \rightarrow -1^+} f(x) =$

h.  $f(2) =$

i.  $\lim_{x \rightarrow 2} f(x) =$

j.  $\lim_{x \rightarrow 1} f(x) =$



6)

a.  $\lim_{x \rightarrow -3} f(x) =$

b.  $f(1) =$

c.  $\lim_{x \rightarrow 1} f(x) =$

d.  $\lim_{x \rightarrow -2^+} f(x) =$

e.  $f(3) =$

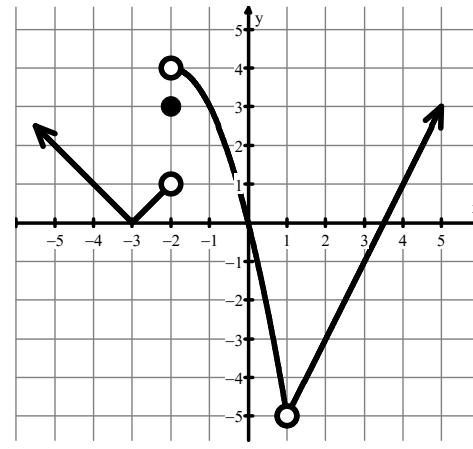
f.  $\lim_{x \rightarrow -2^-} f(x) =$

g.  $\lim_{x \rightarrow -2} f(x) =$

h.  $f(-2) =$

i.  $\lim_{x \rightarrow -1} f(x) =$

j.  $\lim_{x \rightarrow 1^-} f(x) =$



7)

a.  $\lim_{x \rightarrow 3^+} f(x) =$

b.  $f(3) =$

c.  $\lim_{x \rightarrow 0} f(x) =$

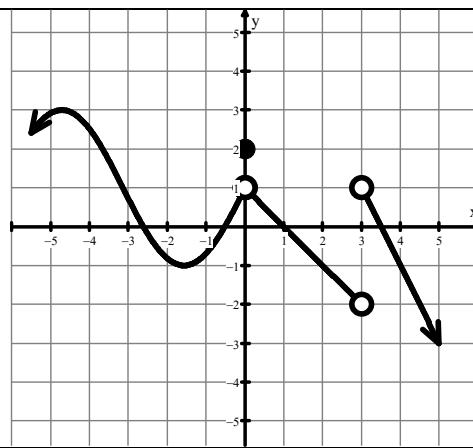
d.  $\lim_{x \rightarrow 3^-} f(x) =$

e.  $f(0) =$

f.  $\lim_{x \rightarrow 3^-} f(x) =$

g.  $\lim_{x \rightarrow 0^+} f(x) =$

h.  $f(1) =$



8)

a.  $\lim_{x \rightarrow -1^-} f(x) =$

b.  $f(2) =$

c.  $\lim_{x \rightarrow 2} f(x) =$

d.  $\lim_{x \rightarrow -1} f(x) =$

e.  $f(4) =$

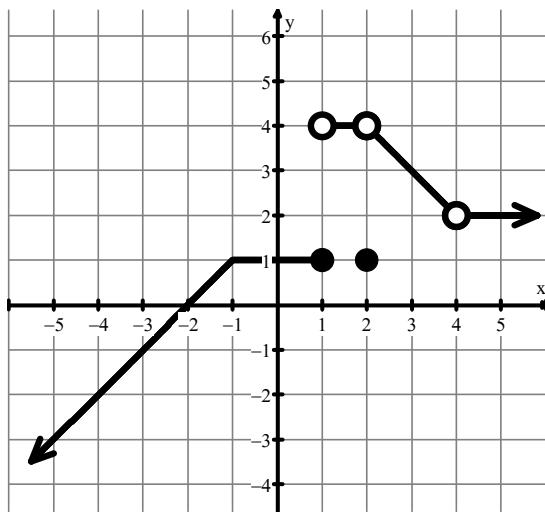
f.  $\lim_{x \rightarrow 1^-} f(x) =$

g.  $\lim_{x \rightarrow -1^+} f(x) =$

h.  $f(1) =$

i.  $\lim_{x \rightarrow 4} f(x) =$

j.  $\lim_{x \rightarrow 1} f(x) =$



9)

a.  $\lim_{x \rightarrow 3^-} f(x) =$

b.  $f(-1) =$

c.  $\lim_{x \rightarrow -3} f(x) =$

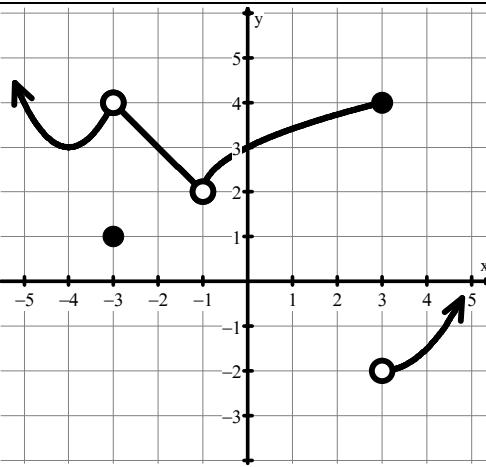
d.  $\lim_{x \rightarrow -1} f(x) =$

e.  $f(-3) =$

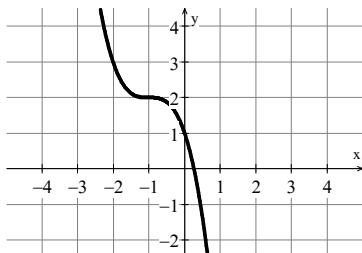
f.  $\lim_{x \rightarrow 3^+} f(x) =$

g.  $f(3) =$

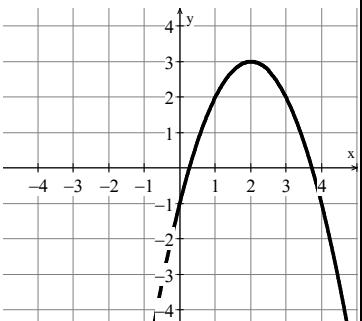
h.  $\lim_{x \rightarrow 0} f(x) =$

**Skillz Review:** Write the function of each graph using  $f(x) = \sqrt{x}$ ,  $f(x) = x^3$ ,  $f(x) = |x|$ , or  $f(x) = x^2$ .

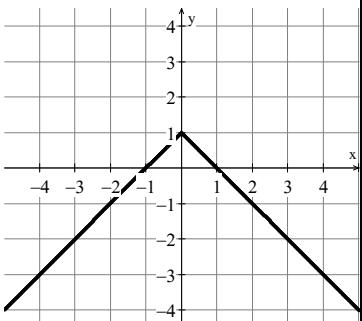
1)  $f(x) =$



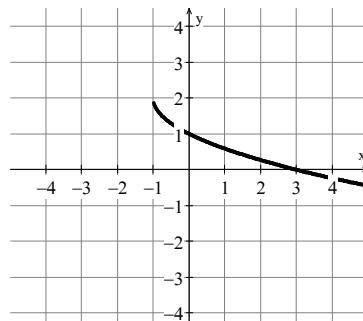
2)  $f(x) =$



3)  $f(x) =$



4)  $f(x) =$



## 2.3 Application and Extension

- 1) Using the graph on the right, give the value of each statement.

a.  $\lim_{x \rightarrow 1^-} f(x) =$

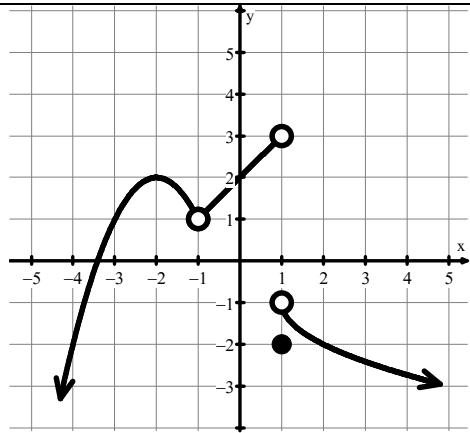
b.  $f(-1) =$

c.  $\lim_{x \rightarrow -1} f(x) =$

d.  $\lim_{x \rightarrow -2} f(x) =$

e.  $f(1) =$

f.  $\lim_{x \rightarrow 1^+} f(x) =$



- 2) A function  $f$  is continuous on  $[-2, 2]$  and some of the values of  $f$  are shown to the right:

$x$	-2	0	2
$f(x)$	3	$b$	4

If  $f$  has only one root,  $r$ , on the closed interval  $[-2, 2]$ , and  $r \neq 0$ , then a possible value of  $b$  is

- a) -3      b) -2      c) -1      d) 0      e) 1

Explain your reasoning in full sentences. (Hint: Draw a picture!!)

- 3) Sketch (freehand) a graph of a function  $f$  that satisfies all of the following conditions:

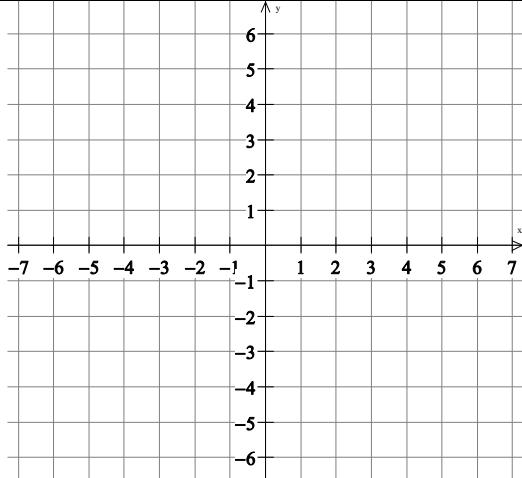
a.  $\lim_{x \rightarrow 3} f(x) = 4$

b.  $\lim_{x \rightarrow 5} f(x) = f(5)$

c.  $f$  is decreasing on  $(-3, 2)$

d.  $f(-3) = -2$

e.  $\lim_{x \rightarrow 2^-} f(x) < \lim_{x \rightarrow 2^+} f(x)$



- 4) Sketch your own function with multiple discontinuities (nonremovable and removable). In the space below, describe your graph using limits for the discontinuities. Also share the intervals on which it is increasing and decreasing.

