3.1 Practice – Discontinuity & Domain

Name: solutions

Pre-Calculus

For 1 – 9, **find** and **classify** each discontinuity.

1. $f(x) = \frac{x}{x-3}$ $= 0$

$\boxed{x=3 \text{ V.A.}}$

2. $g(x) = \sqrt{9+4x}$

Continuous on its domain.

(No discontinuities!)

3. $h(x) = \frac{\cancel{x-5}}{x^2-4x-5}$

$(x+1)(\cancel{x-5}) = 0$

$\boxed{x=-1 \text{ V.A.}}$ $\boxed{x=5 \text{ Hole}}$

4. $a(x) = \frac{2x^2-x-1}{2x^2+5x-3}$

$\frac{(2x+1)(x-2)}{(2x-1)(x+3)} = 0$

$\boxed{x=\frac{1}{2} \text{ V.A.}}$ $\boxed{x=-3 \text{ V.A.}}$

5. $w(x) = \frac{5x+15}{3}$ $= 0$

Not possible.

$\boxed{\text{Continuous on its domain.}}$

6. $f(x) = \frac{\cancel{3x+4}}{9x^2-16}$ $= 0$

$(\cancel{3x+4})(3x-4) = 0$

$\boxed{x=-\frac{4}{3} \text{ hole}}$ $\boxed{x=\frac{4}{3} \text{ V.A.}}$

Skillz Review: Solve or evaluate.

1. $\sqrt{-32}$

$i\sqrt{16}\sqrt{2}$

$\boxed{4i\sqrt{2}}$

2. $x^2 = -75$

$x = \pm\sqrt{-75}$

$x = \pm i\sqrt{25}\sqrt{3}$

$\boxed{x = \pm 5i\sqrt{3}}$

3. $(x-3)^2 = 25$

$x-3 = \pm 5$

$x = 3 \pm 5$

$\boxed{x = 8 \text{ or } -2}$

4. $(x-5)^2 = -17$

$x-5 = \pm\sqrt{-17}$

$\boxed{x = 5 \pm i\sqrt{17}}$

7. $h(t) = \dfrac{3t^2 + t}{t^3 + 3t^2 - 28t}$

$\dfrac{\cancel{t}(3t+1)}{\cancel{t}(t+7)(t-4)} = 0$

$\boxed{t=0 \text{ hole}}$ $\boxed{t=-7 \text{ V.A.}}$ $\boxed{t=4 \text{ V.A.}}$

8. $a(x) = \dfrac{6x^2 + 19x - 7}{10x^2 + 37x + 7}$

$\dfrac{(3x-1)\cancel{(2x+7)}}{\cancel{(2x+7)}(5x+1)} = 0$

$\boxed{x=-\tfrac{7}{2} \text{ hole}}$ $\boxed{x=-\tfrac{1}{5} \text{ V.A.}}$

9. $f(x) = \dfrac{2}{x^2 + 4}$ $=0$

$x^2 = -4$

imaginary solutions

$\boxed{\text{Continuous on its domain.}}$

For 10 – 21, identify the domain of each function. (use inequality notation)

10. $w(x) = \dfrac{\sqrt{2x-5}}{3}$

$2x-5 \geq 0$ $\quad$ $3 \neq 0$

$2x \geq 5$ $\quad$ Always true

$\boxed{x \geq \tfrac{5}{2}}$

11. $s(t) = \dfrac{5}{\sqrt{4t-8}}$

$4t-8 \geq 0$ $\quad$ $\sqrt{4t-8} \neq 0$

$4t \geq 8$ $\quad$ $4t-8 \neq 0$

$t \geq 2$ $\quad$ $t \neq 2$

$\boxed{t > 2}$

12. $f(x) = \dfrac{x}{\sqrt{36-6x}}$

$36-6x \geq 0$ $\quad$ $\sqrt{36-6x} \neq 0$

$-6x \geq -36$ $\quad$ $36-6x \neq 0$

$x \leq 6$ $\quad$ $x \neq 6$

$\boxed{x < 6}$

13. $g(x) = \dfrac{x+7}{x^2 - 2x - 15}$

$(x-5)(x+3) \neq 0$

$x \neq 5, \quad x \neq -3$

$\boxed{\mathbb{R}, x \neq -3, 5}$

14. $v(t) = \dfrac{2t}{t\sqrt{t+6}}$

$t \neq 0$ $\quad$ $\sqrt{t+6} \neq 0$ $\quad$ $t+6 \geq 0$

$t+6 \neq 0$ $\quad$ $t \geq -6$

$t \neq -6$

$\boxed{t > -6, \ t \neq 0}$

15. $g(w) = \dfrac{7}{5 - \sqrt{w}}$

$5 - \sqrt{w} \neq 0$ $\quad$ $w \geq 0$

$-\sqrt{w} \neq -5$

$w \neq 25$

$\boxed{w \geq 0, \ w \neq 25}$

16. $s(t) = \sqrt[3]{3t-9}$

Odd

$\boxed{\text{Domain: All real numbers}}$

17. $g(x) = \dfrac{x}{|x| - 3}$

$|x| - 3 \neq 0$

$|x| \neq 3$

$x \neq \pm 3$

$\boxed{\mathbb{R}, x \neq -3, 3}$

18. $h(t) = \dfrac{\sqrt{1-t}}{t-3}$

$t-3 \neq 0$ $\quad$ $1-t \geq 0$

$t \neq 3$ $\quad$ $-t \geq -1$

$t \leq 1$

$\boxed{t \leq 1}$

19. $a(t) = (t-4)(\sqrt{t})$

$\boxed{t \geq 0}$

20. $g(x) = x^3 + 7x^2 + 12x$

No even radicals.
No variables in a denominator.

Therefore...

$\boxed{\mathbb{R}}$

21. $h(t) = \dfrac{t^2 - t}{5t^3 - 7t^2 + 2t} \neq 0$

$t(5t-2)(t-1) \neq 0$

$t \neq 0 \quad t \neq \frac{2}{5} \quad t \neq 1$

$\boxed{\mathbb{R},\ t \neq 0, \frac{2}{5}, 1}$

For 22 – 27, identify the domain of each function AND classify each discontinuity.

22. $w(x) = \dfrac{8x+12}{4} \neq 0$

Domain: $\mathbb{R}$

Discontinuities:
continuous on its domain

23. $f(x) = \dfrac{8x-5}{64x^2 - 25}$

$(8x-5)(8x+5) = 0$

$x \neq \frac{5}{8} \quad x \neq -\frac{5}{8}$

Domain:
$\mathbb{R},\ x \neq -\frac{5}{8}, \frac{5}{8}$

Discontinuities:
hole at $x = \frac{5}{8}$
V.A. at $x = -\frac{5}{8}$

24. $h(x) = \dfrac{x+1}{x^2 - 5x - 6}$

$(x+1)(x-6) \neq 0$

$x \neq -1 \quad x \neq 6$

Domain:
$\mathbb{R},\ x \neq -1, 6$

Discontinuities:
hole at $x = -1$
V.A. at $x = 6$

25. $v(x) = \dfrac{3x}{x\sqrt{x+9}}$

$x \neq 0 \quad \sqrt{x+9} \neq 0 \quad x+9 \geq 0$

$x+9 \neq 0 \quad x \geq -9$

$x \neq -9$

Domain:
$x > -9,\ x \neq 0$

Discontinuities:
hole at $x = 0$

26. $g(x) = \dfrac{\sqrt{5-x}}{x-8}$

$x - 8 \neq 0 \quad 5 - x \geq 0$

$x \neq 8 \quad -x \geq -5$

$x \leq 5$

Domain:
$x \leq 5$

Discontinuities: none
Cont. on its domain

27. $f(x) = \dfrac{1}{x^2+1}$

$x^2 + 1 \neq 0$

$x^2 \neq -1$ imaginary?

Domain:
$\mathbb{R}$

Discontinuities: none
Continuous on its domain.