

5.2 Practice – Poly Division and Factor Theorems

Name: Solutions

Pre-Calculus

For 1-3, use long division to divide the polynomials.

1. $(2x^2 - 7x + 4) \div (x - 2)$

$$\begin{array}{r} 2x - 3 \\ x - 2 \overline{) 2x^2 - 7x + 4} \\ \underline{-(2x^2 - 4x)} \\ -3x + 4 \\ \underline{-(-3x + 6)} \\ -2 \end{array}$$

$$2x - 3 - \frac{2}{x - 2}$$

2. $(y^2 - 9) \div (y + 1)$

$$\begin{array}{r} y - 1 \\ y + 1 \overline{) y^2 + 0y - 9} \\ \underline{-(y^2 + y)} \\ -y - 9 \\ \underline{-(-y - 1)} \\ -8 \end{array}$$

$$y - 1 - \frac{8}{y + 1}$$

3. $(9x^5 - 3x^3 + 21x^2 - 2x + 4) \div (3x^2 + 1)$

$$\begin{array}{r} 3x^3 - 2x + 7 \\ 3x^2 + 0x + 1 \overline{) 9x^5 + 0x^4 - 3x^3 + 21x^2 - 2x + 4} \\ \underline{-(9x^5 + 0x^4 + 3x^3)} \\ -6x^3 + 21x^2 - 2x + 4 \\ \underline{-(-6x^3 + 0x^2 - 2x)} \\ 21x^2 + 0x + 4 \\ \underline{-(21x^2 + 0x + 7)} \\ -3 \end{array}$$

$$3x^3 - 2x + 7 - \frac{3}{3x^2 + 1}$$

For 4-6, use synthetic division to divide the polynomials

4. $(x^2 + 3x - 3) \div (x - 3)$

$$\begin{array}{r} 3 \overline{) 1 \quad 3 \quad -3} \\ \underline{3 \quad 18} \\ 1 \quad 6 \quad 15 \end{array}$$

$$x + 6 + \frac{15}{x - 3}$$

5. $(7x^6 + 50x^3 + 20x) \div (x + 2)$

$$\begin{array}{r} -2 \overline{) 7 \quad 0 \quad 0 \quad 50 \quad 0 \quad 20 \quad 0} \\ \underline{14 \quad 28 \quad -56 \quad 12 \quad -24 \quad 8} \\ 7 \quad 14 \quad 28 \quad -6 \quad 12 \quad -4 \quad 8 \end{array}$$

$$7x^5 - 14x^4 + 28x^3 - 6x^2 + 12x - 4 + \frac{8}{x + 2}$$

6. $(3x^4 - 4x^2 + 1) \div (x + 1)$

$$\begin{array}{r} -1 \overline{) 3 \quad 0 \quad -4 \quad 0 \quad 1} \\ \underline{-3 \quad 3 \quad 1 \quad -1} \\ 3 \quad -3 \quad -1 \quad 1 \quad 0 \end{array}$$

$$3x^3 - 3x^2 - x + 1$$

7. If $(x + 1)$ is a factor of $2x^5 + 2x^4 - 5x^3 - 5x^2 - 3x - 3$, what are all the factors of $f(x)$.

$$\begin{array}{r} -1 \overline{) 2 \quad 2 \quad -5 \quad -5 \quad -3 \quad -3} \\ \underline{-2 \quad 0 \quad 5 \quad 0 \quad 3} \\ 2 \quad 0 \quad -5 \quad 0 \quad -3 \quad 0 \\ 2x^4 - 5x^2 - 3 \end{array}$$

$$(2x^2 + 1)(x^2 - 3)(x + 1)$$

8. Is $(x - 11)$ a factor of $(3x^4 - 33x^3 - 17x^2 + 187x - 11)$?

$$\begin{array}{r} 11 \overline{) 3 \quad -33 \quad -17 \quad 187 \quad -11} \\ \underline{33 \quad 0 \quad -187} \\ 3 \quad 0 \quad -17 \quad 0 \quad -11 \end{array}$$

No!

9. If $(3x + 5)$ is a factor of $6x^3 + 31x^2 + 23x - 20$, what are all the factors of $f(x)$.

$$\begin{array}{r|rrrr} -\frac{5}{3} & 6 & 31 & 23 & -20 \\ & \cancel{30} & -10 & -35 & 20 \\ \hline & 6 & 21 & -12 & 0 \end{array}$$

$$\therefore \rightarrow 2x^2 + 7x - 4$$

$$(2x-1)(x+4) \text{ and } (3x+5)$$

10. Is $(n + 1)$ a factor of $(6n^3 + 6n^2)$?

$$\begin{array}{r|rrrr} -1 & 6 & 6 & 0 & 0 \\ & \cancel{6} & -6 & 0 & 0 \\ \hline & 6 & 0 & 0 & 0 \end{array}$$

Yes!

For 11-14, a zero of the function is given. Find ALL the zeros of the function.

11. $f(x) = 4x^3 - 25x^2 - 154x + 40$; $f(10) = 0$

$$\begin{array}{r|rrrr} 10 & 4 & -25 & -154 & 40 \\ & \cancel{40} & 40 & 150 & -40 \\ \hline & 4 & 15 & -4 & 0 \end{array}$$

$$4x^2 + 15x - 4 = 0$$

$$(4x-1)(x+4) = 0$$

$$4x-1=0 \quad x+4=0$$

$$x = 10, \frac{1}{4}, -4$$

12. $f(x) = 3x^3 + 34x^2 + 72x - 64$; $f(-4) = 0$

$$\begin{array}{r|rrrr} -4 & 3 & 34 & 72 & -64 \\ & \cancel{12} & -12 & -88 & 64 \\ \hline & 3 & 22 & -16 & 0 \end{array}$$

$$3x^2 + 22x - 16 = 0$$

$$(3x-2)(x+8) = 0$$

$$x = -4, \frac{2}{3}, -8$$

13. $f(x) = x^5 + 2x^4 + 7x^3 + 14x^2 + 6x + 12$
 $f(-2) = 0$

$$\begin{array}{r|rrrrrr} -2 & 1 & 2 & 7 & 14 & 6 & 12 \\ & \cancel{2} & -2 & 6 & -14 & 0 & -12 \\ \hline & 1 & 0 & 7 & 0 & 6 & 0 \end{array}$$

$$x^3 + 7x^2 + 6 = 0$$

$$(x^2+6)(x+1) = 0$$

$$x^2 = -6 \quad x = -1$$

$$x = -2, \pm i\sqrt{6}, -1$$

14. $f(x) = x^3 - 125$; $f(5) = 0$

$$\begin{array}{r|rrrr} 5 & 1 & 0 & 0 & -125 \\ & \cancel{5} & 5 & 25 & 125 \\ \hline & 1 & 5 & 25 & 0 \end{array}$$

$$x^2 + 5x + 25 = 0$$

$$x = \frac{-5 \pm \sqrt{25 - 4(1)25}}{2}$$

$$x = 5, \frac{-5 \pm 5i\sqrt{3}}{2}$$

For 15-16, use the graph of the function to determine at least one zero, then find the exact values of all the zeros using the Factor Theorem.

15. $f(x) = 3x^4 + 16x^3 - 8x^2 - 112x - 91$

$$\begin{array}{r|rrrrr} -1 & 3 & 16 & -8 & -112 & -91 \\ & & -3 & 13 & 21 & 91 \\ \hline & 3 & 13 & -21 & -91 & 0 \end{array}$$

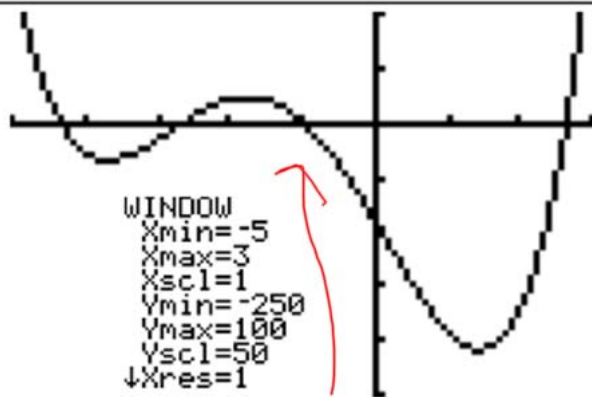
$$3x^3 + 13x^2 - 21x - 91 = 0$$

$$x^2(3x+13) - 7(3x+13) = 0$$

$$(x^2 - 7)(3x + 13) = 0$$

$$x^2 = 7 \quad 3x = -13$$

$$x = -1, \pm\sqrt{7}, -\frac{13}{3}$$



$$x = -1$$

16. $f(x) = 10x^3 - 31x^2 - 76x + 160$

$$\begin{array}{r|rrrr} 4 & 10 & -31 & -76 & 160 \\ & & 40 & 36 & -160 \\ \hline & 10 & 9 & -40 & 0 \end{array}$$

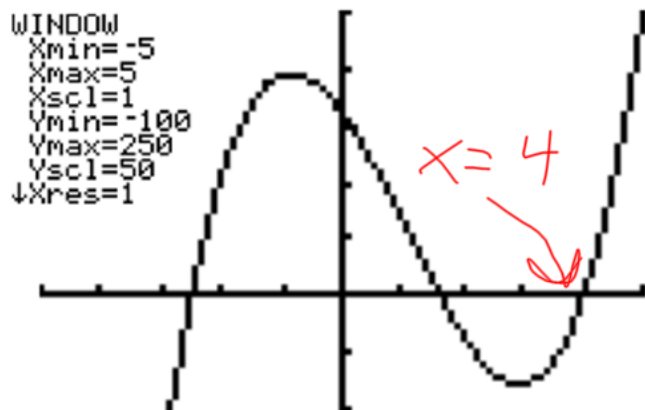
$$10x^2 + 9x - 40 = 0$$

$$10x^2 + 25x - 16x - 40 = 0$$

$$5x(2x+5) - 8(2x+5) = 0$$

$$(2x+5)(5x-8) = 0$$

$$x = 4, -\frac{5}{2}, \frac{8}{5}$$



$$x = 4$$

For 17-20, one zero is given of $f(x)$. List one other zero.

17. $13 - 25i$

$$13 + 25i$$

18. $10i$

$$-10i$$

19. $3i + 1$

$$-3i + 1$$

20. $\sqrt{7} + 14i$

$$\sqrt{7} - 14i$$

Skills Review: Find the x- and y-intercepts for each function. SHOW ALL WORK!

1. $12x - 5y = 60$

x-int:

$$12x - 5(0) = 60$$

$$12x = 60$$

$$x = 5$$

y-int:

$$12(0) - 5y = 60$$

$$-5y = 60$$

$$y = -12$$

2. $f(x) = \frac{x^2 + x - 6}{4x - 4}$

x-int:

$$0 = \frac{x^2 + x - 6}{4x - 4}$$

$$0 = x^2 + x - 6$$

$$0 = (x+3)(x-2)$$

$$x = -3, 2$$

y-int:

$$y = \frac{0+0-6}{4(0)-4}$$

$$y = \frac{-6}{-4}$$

$$y = \frac{3}{2}$$