

7.3 Exponential and Logarithmic Problem Solving Practice

Solve, rounding to three significant digits.

1. $10^{-x} = 0.0347$

$$\log 10^{-x} = \log 0.0347$$

$$-x = \log 0.0347$$

$$x = 1.460$$

3. $e^x = 3.65$

$$\ln e^x = \ln 3.65$$

$$x = 1.295$$

5. $\log_5 x = 2$

$$x = 5^2$$

$$x = 25$$

Just A#

7. $\log 5 + \log x = 2$

$$\log x = 2 - \log 5$$

$$\log x = 1.301$$

$$10^{\log x} = 10^{1.301}$$

$$x = 20$$

9. $\log(5 - 2x) = \log(3x + 1)$

$$5 - 2x = 3x + 1$$

$$4 = 5x$$

$$x = 4/5$$

11. $3 - \log(x + 3) = 5$

$$-\log(x + 3) = 2$$

$$\log(x + 3) = -2$$

$$10^{\log(x + 3)} = 10^{-2}$$

$$x + 3 = .01$$

$$x = -2.99$$

2. $10^{3x+1} = 92$

$$\log 10^{3x+1} = \log 92$$

$$3x+1 = 1.964$$

$$x = 0.321$$

4. $e^{2x-1} + 68 = 207$

$$\ln e^{2x-1} = \ln 139$$

$$2x-1 = 4.934$$

$$x = 2.967$$

6. $80e^{0.045x} = 240$

$$\ln e^{0.045x} = \ln 3$$

$$0.045x = 1.0986$$

$$x = 24.414$$

8. $\log x + \log(x - 3) = 1$

$$10^{\log x(x-3)} = 10^1$$

$$x(x-3) = 10$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$x = 5$ OR $x = -2$ (extraneous)

10. $\ln x = \ln(2x - 1) - \ln(x - 2)$

$$e^{\ln x} = e^{\ln \frac{2x-1}{x-2}}$$

$$x = \frac{2x-1}{x-2}$$

$$x^2 - 2x = 2x - 1$$

$$x^2 - 4x + 1 = 0$$

$x = 0.268$ is extraneous

$$x = 3.732$$

12. $3^{x+1} = 28$

$$\log_3 3^{x+1} = \log_3 28$$

$$x+1 = \log_3 28$$

$$x = \log_3 28 - 1$$

$$x = 2.033$$

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Application 7.3

1. $\ln x^6 = 12$

$$e \quad (x^6)^{\frac{1}{6}} = (e^{12})^{\frac{1}{6}}$$

$$x = e^2 = 7.389$$

2. $\log(t-4) = -1$

$$10^{\log(t-4)} = 10^{-1}$$

$$t-4 = 0.1$$

$$t = 4.1$$

3. A hard-boiled egg at temperature 96°C is placed in 16°C water to cool. Four minutes later the temperature of the egg is 45°C . Use Newton's Law of Cooling to find k .

$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

$$45 = 16 + (96 - 16)e^{-k(4)}$$

$$45 = 16 + 80e^{-4k}$$

$$29 = 80e^{-4k}$$

$$\frac{29}{80} = e^{-4k}$$

$$\ln \frac{29}{80} = \ln e^{-4k}$$

$$\ln \frac{29}{80} = -4k$$

$$\frac{\ln \frac{29}{80}}{-4} = k$$

$$k = .2537$$

Now, use k to find the time when $T = 20^\circ\text{C}$

$$20 = 16 + (96 - 16)e^{-.2537t}$$

$$4 = 80e^{-.2537t}$$

$$\ln \frac{4}{80} = \ln e^{-.2537t}$$

$$\frac{\ln \frac{4}{80}}{-.2537} = t$$

$$t = 11.8 \text{ min}$$

4. There were 12 earthquakes recorded world-wide in 2008 with a magnitude at least 7.0.

$$M = \frac{2}{3} \log \frac{E}{E_0}$$

$$E_0 = 10^{4.4}$$

a. How much energy is released by a magnitude 7.0 earthquake?

$$7.0 = \frac{2}{3} \log \frac{E}{10^{4.4}}$$

$$10^{10.5} = \log \frac{E}{10^{4.4}}$$

$$3.16 \times 10^{10} = \frac{E}{10^{4.4}}$$

$$E = 7.94 \times 10^{14} \text{ joules}$$

b. The total average daily consumption of energy for the entire United States in 2006 was 2.88×10^{14} joules. How many days could the energy released by a magnitude 7.0 earthquake power the United States?

$$2.758 \text{ days!}$$

$$\frac{7.94 \times 10^{14}}{2.88 \times 10^{14}}$$

5. A new liquid-fueled rocket has a weight ratio of $\frac{w_t}{w_b} = 6.2$ and an exhaust velocity of $c = 5.2 \text{ km/sec}$. What is the velocity at burnout?

$$V = c \ln \frac{w_t}{w_b}$$

$$= (5.2 \text{ km}) \ln(6.2)$$

$$V = 9.48765 \text{ km/sec}$$

6. Generally, an earthquake requires a magnitude of over 5.6 on the Richter scale to inflict serious damage. How many more times powerful was this earthquake than the one in Columbia in 1906, which registered an 8.6?

$$\frac{10^{8.6}}{10^{5.6}} = 1000 \text{ times!}$$

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7. How many times larger is the sound intensity from Sully's classroom before Sully Law is enacted (8.5×10^{-4}) compared to when after Sully law is enacted (5.2×10^{-10}).

$$\frac{10 \log\left(\frac{8.5 \times 10^{-4}}{5.2 \times 10^{-10} \cdot 10^{-12}}\right)}{10 \log\left(\frac{5.2 \times 10^{-10}}{10^{-12}}\right)} \approx 3.2877 \text{ times}$$

8. The earliest mechanical clocks appeared around 1350 in Europe, and would gain or lose an average of 30 minutes per day. After that, accuracy doubled every 30 years. Find the predicted accuracy of clocks in the year:

a. 1700 $E = 30 \left(\frac{1}{2}\right)^{\frac{1700-1350}{30}} = 30 \left(\frac{1}{2}\right)^{\frac{350}{30}}$
 Error = .009 m/day
 ERROR CUT IN HALF

b. 2000
 Error = $30 \left(\frac{1}{2}\right)^{\frac{2000-1350}{30}} = 9.01 \times 10^{-6}$ m/day
 This is how much error clocks have in yr. 2000

1700 - 1350 = 350
 350 / 30 = 11.666
 This is how many "Doublings"

9. In 2003, Japanese scientists announced an effort to bring back the long-extinct Woolly-Mammoth using cloning techniques. They found a well preserved specimen in Siberia. Nearby plant specimens were found to have 28.9% of the amount of carbon-14 in a living sample. How old is the specimen found in Siberia?

$$1000 = \frac{\ln\left(\frac{100}{28.9}\right)}{-0.000124} = t$$

about 10,010 yrs old

$\frac{28.9}{100} = \frac{100}{28.9} e^{-0.000124 t}$

10. At what rate compounded continuously will \$1000 have to be invested to amount to \$2500 in 10 years?

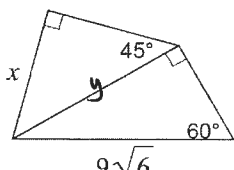
$$A = Pe^{rt}$$

$$2500 = 1000 e^{r(10)}$$

$$\ln\left(\frac{2500}{1000}\right) = 10r$$

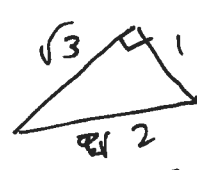
$$r = .0916 \text{ or } 9.16\%$$

Quick Review. Find x.

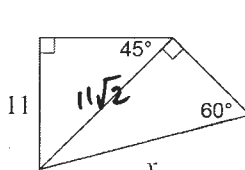
11. 

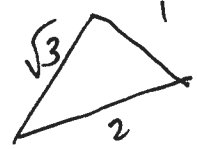
$\frac{9\sqrt{6}}{2} = \frac{y}{\sqrt{3}}$

$x = \frac{27}{2}$



$2y = 9\sqrt{18}$
 $2y = 27\sqrt{2}$
 $y = \frac{27}{2}\sqrt{2}$

12. 



$\frac{11\sqrt{2}}{\sqrt{3}} = \frac{x}{2}$

$\frac{22\sqrt{2}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{22\sqrt{6}}{3}$

$22\sqrt{2} = (\sqrt{3})x$

$x = \frac{11}{3}\sqrt{6}$