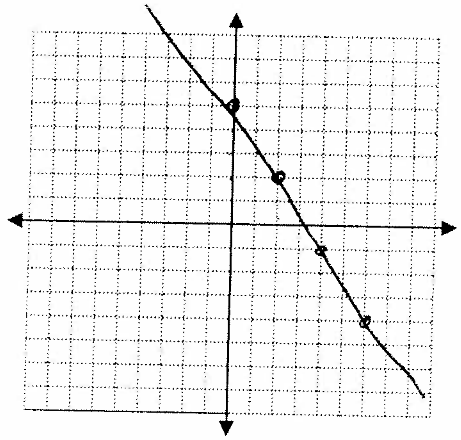
NAME:________________________________ DATE:____________

SEMESTER 1 EXAM REVIEW

Unit 1: Intro to Pre-Calc

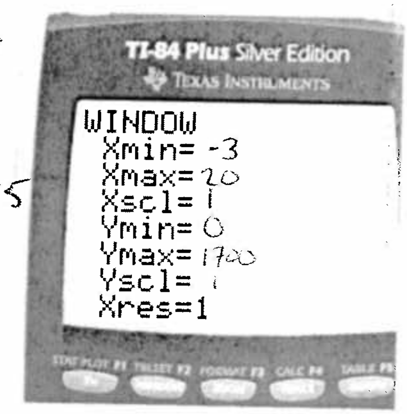
1. Linear Functions

Slope Intercept Form	**Standard Form**	**Point Slope Form**
a. Write the equation of the line in slope intercept form that is perpendicular to $y = 2x - 5$ and contains (-50,10) m=2 ⊥ = $-\frac{1}{2}$ $y - y_1 = m(x - x_1)$ $y - 10 = -\frac{1}{2}(x + 50)$ $y - 10 = -\frac{1}{2}x - 25$ +10 +10 $\boxed{y = -\frac{1}{2}x - 15}$	b. Graph $3x + 2y = 10$ $2y = -3x + 10$ $y = -\frac{3}{2}x + 5$ 	c. Write the equation of the line in point slope form that contains the points (-50,-49) and (35,53) $m = \frac{53+49}{35+50} = \frac{102}{85} = \frac{6}{5}$ $\boxed{y + 49 = \frac{6}{5}(x + 50)}$ or $\boxed{y - 53 = \frac{6}{5}(x - 35)}$

2. Regression (Best Fit Line/Curve)
The following table gives the number of motor vehicle thefts (in thousands) in the U.S. for the years 1983 - 1993. $x = 1$ represents 1983. Use the regression capabilities of your calculator to fit a cubic model to this data.

Year	1	3	4	7	8	9	10	11
Vehicle Thefts	1008	1103	1224	1565	1636	1662	1611	1561

a. Graph the data with a friendly window. Record here →



b. Use regression and write the equation of your model. (Round to three decimal places)

$V(t) = -2.29x^3 + 34.13x^2 - 50.29x + 1021.85$

c. What does $V(5.5)$ mean? Find it. 1396.05

d. Find the time(s) in which there will be1400 thousand auto thefts.

5.5 yrs → 1988/1989   11.85 yrs → 1994/1995

e. What does the y-intercept mean in this situation?

# of thefts in 1982

f. Predict the auto thefts in 1995. x = 12   so 1369.7 thefts

3. Factoring Basics: Solve the following by factoring.

a. $x^2 - 9x = 0$
$x(x - 9) = 0$
$x = 0$ or $x - 9 = 0$
+9 +9
$x = 9$

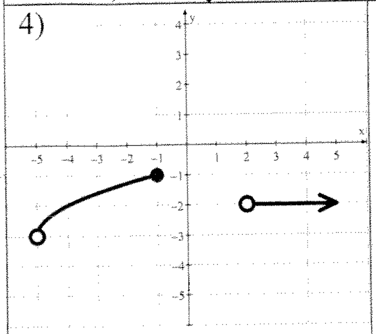
b. $x^2 - 9x - 112 = 0$
$(x + 7)(x - 16) = 0$
$x + 7 = 0$ or $x - 16 = 0$
$x = -7$ $x = 16$

c. $2x^2 - 17x = -35$
x 70
+ -17
$2x^2 - 17x + 35 = 0$
$(2x^2 - 10x) + (-7x + 35) = 0$
$2x(x - 5) + -7(x - 5) = 0$
$(2x - 7)(x - 5) = 0$
$2x - 7 = 0$
$2x = 7$
$\boxed{x = 7/2}$
$x - 5 = 0$

Unit 2: Functions and Limits

For 4-5, identify the domain and range of each function. Use both interval notation and inequality notation.

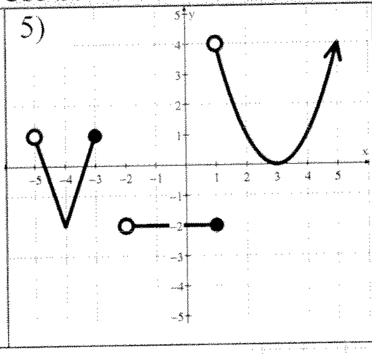
4)



Domain:
Interval: $(-5,\cancel{0}-1] \cup (2,\infty)$
Inequality: $-5<x\le -1$ or ~~and~~ $x>2$

Range:
Interval: $(-3,-1]$
Inequality: $-3<y\le -1$

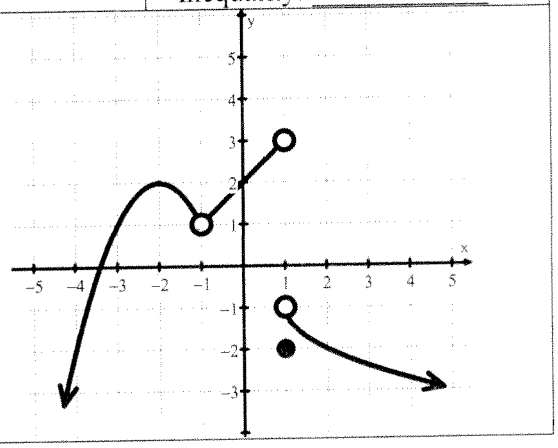
5)



Domain:
Interval: $(-5,-3] \cup (-2,\infty)$
Inequality: $-5<x\le -3$ OR $x>-2$

Range:
Interval: $[-2,\infty)$
Inequality: $y\ge -2$

6) Using the graph on the right, give the value of each statement.



a. $\lim\limits_{x\to 1^-} f(x) =$ 3

b. $f(-1) =$ DNE

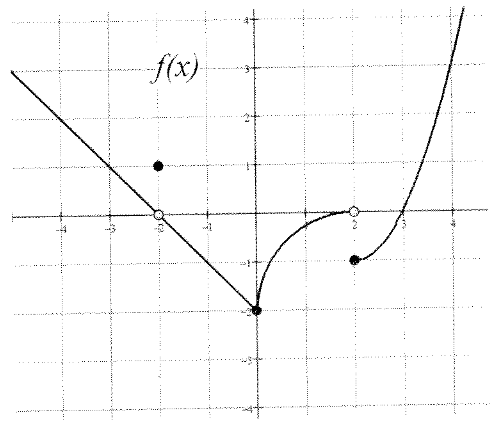
c. $\lim\limits_{x\to -1} f(x) =$ 1

d. $\lim\limits_{x\to -2^-} f(x) =$ 2

e. $f(1) =$ -2

f. $\lim\limits_{x\to 1^+} f(x) =$ -1

For questions 7-14, refer to the graph of *f*(*x*)



7. State the intervals where $f(x)$ is continuous.
$(-\infty,-2)$
$(-2,2)$ $(2,\infty)$

8. State the values of x where the function is discontinuous and label them as removable or non-removable discontinuities. $x=-2$ and $x=2$
Removable

9. $\lim\limits_{x\to 0} f(x) =$ -2

10. $\lim\limits_{x\to 2^+} f(x) =$ -1

11. $\lim\limits_{x\to 2^-} f(x) =$ 0

12. $\lim\limits_{x\to 2} f(x) =$ DNE

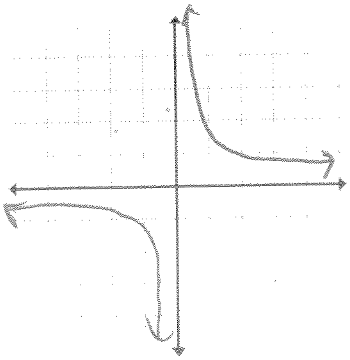
13. $f(0) =$ -2

14. $\lim\limits_{x\to -2} f(x) =$ 0

For 15-16, graph the function and determine if it has points of discontinuity. If there is a discontinuity, tell what type of discontinuity it is and its x-value. Clearly mark all asymptotes with a dotted line.

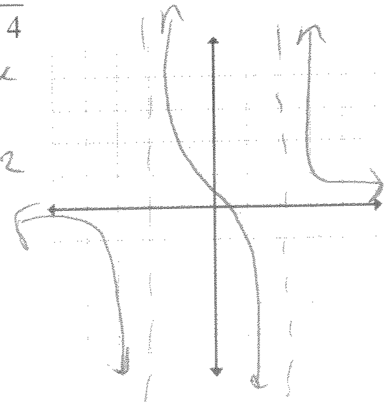
15. $f(x) = \dfrac{2}{x}$

NON REMOVABLE
DISCONTINUITY
at $x=0$
(Vert. Asym.)



16. $f(x) = \dfrac{x}{x^2-4}$

NON Removable
Discontinuity
at $x=-2,2$
(Vert. Asy)



Unit 3: Function Analysis

For 17-19, find the domain of the given function. Use interval notation.

17. $f(x)=\sqrt{10-x}$

$10-x \geq 0$

$10 \geq x$

$\boxed{(-\infty, 10]}$

18. $f(x)=\dfrac{x}{x-5}$

$x \neq 5$

$\boxed{(-\infty,5)\cup(5,\infty)}$

19. $f(x)=\dfrac{\sqrt{x-2}}{(x+7)(x-8)}$ $x-2\geq 0;\ x \geq 2$

$x \neq -7$ $x \neq 8$

$\boxed{[2,8)\cup(8,\infty)}$

For 20-21, find the range of the function. Use interval notation.

20. $f(x)=(x-4)^2+4$

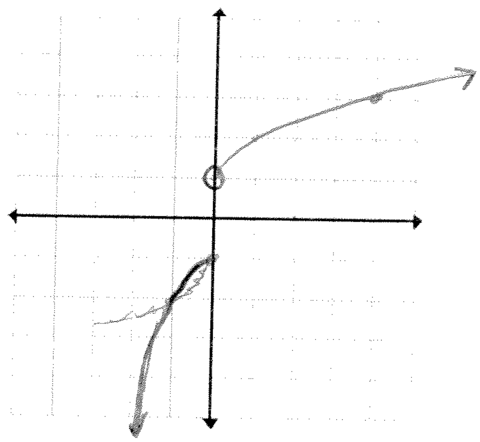
(4,4)

$\boxed{[4,\infty)}$

21. $f(x)=\sqrt{9+x}$

$\boxed{[0,\infty)}$

22. Sketch the piecewise function $f(x)=\begin{cases} x^3-1 & x\leq 0 \\ \sqrt{x}+1 & x>0 \end{cases}$

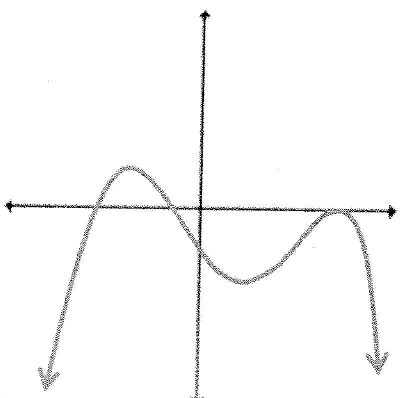


a. $f(2)=$ $\boxed{\sqrt{2}+1}$

b. $f(-2)=(-2)^3-1=\boxed{-9}$

c. $f(0)=(0)^3-1=\boxed{-1}$

23. Label all local and absolute maximums and minimums.

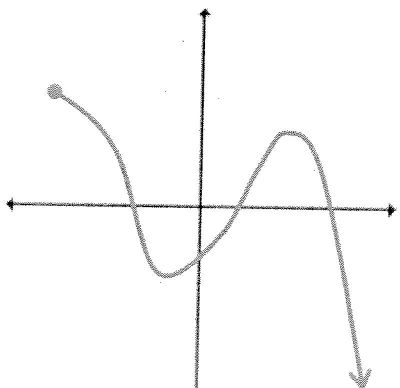


Abs max @ $x=-2$

Relative max @ $x=3$.

Relative Min @ $x=1$

b.

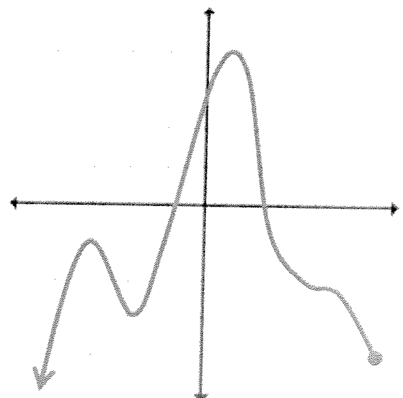


Abs max @ $x=-4$

Rel max @ $x=2$

Rel min @ $x=-1$

c.



Abs max @ $x\approx .7$

Rel max @ $x=-3$

Rel min @ $x=-2$

Unit 4: Function Algebra

24. $f(x)=3x+11$ and $g(x)=5x-1$

$f\cdot g=(3x+11)(5x-1)$

$15x^2-3x+55x-11$

$\boxed{15x^2+52x-11}$

$\dfrac{f}{g}=\boxed{\dfrac{3x+11}{5x-1}}$

25. $f(x)=4x^2+2x+3;\ g(x)=2x-4$

$f\circ g=4(2x-4)^2+2(2x-4)+3$

$=4(2x-4)(2x-4)+4x-8+3$

$=4(4x^2-16x+16)+4x-5$

$=16x^2-\cancel{64x+64}+4x-5$

$\boxed{=16x^2-60x+59}$

For 26, confirm that f and g are inverses by showing the $f(g(x)) = x$

26. $f(x) = 2x + 9$ and $g(x) = \frac{x-9}{2}$

$f(g(x)) = 2\left(\frac{x-9}{2}\right) + 9$

$= x - 9 + 9$

$\boxed{f(g(x)) = x}$

For 27-32, if $f(x) = 2x - 5$ and the $g(x) = x^2 + 2x - 3$, find the following…

27. $f(2)$

$2(2) - 5$

$4 - 5$

$f(2) = -1$

28. $g(-2)$

$(-2)^2 + 2(-2) - 3$

$4 - 4 - 3$

$g(-2) = -3$

29. $f(g(0))$

$g(0) = 0^2 + 2(0) - 3$

$g(0) = -3$

$f(-3) = 2(-3) - 5$

$= -6 - 5$

$\boxed{f(g(0)) = -11}$

30. $f - g$

$2x - 5 - (x^2 + 2x - 3)$

$f - g = \boxed{-x^2 - 2}$

31. $f(x + h)$

$f(x+h) = 2(x+h) - 5$

$\boxed{f(x+h) = 2x + 2h - 5}$

32. $(f + g)(2)$

$(f+g)(2) = 2x - 5 + (x^2 + 2x - 3)$

$(f+g)(2) = x^2 + 4x - 8$

$= 2^2 + 4(2) - 8$

$= 4 + 8 - 8$

$\boxed{(f+g)(2) = 4}$

33. Is $f(x) = \frac{x+1}{x^2-1}$ even, odd, or neither. Justify your answer!

$f(-x) = \frac{-x+1}{(-x)^2-1} = \frac{-x+1}{x^2-1} = -\frac{x-1}{x^2-1}$ so NEITHER because $f(-x) \neq -f(x)$ or $f(-x) \neq f(x)$

Transformations

34. $y = 2(x - 5)^3 - 4$

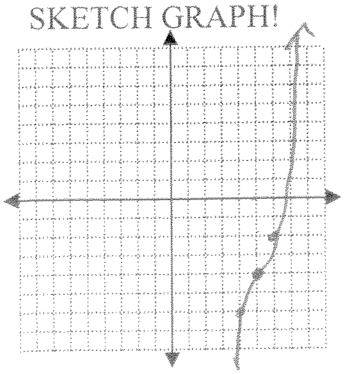
Name function: CUBIC

Translation: RIGHT 5, DOWN 4

Scale: Vertical STRETCH OF 2

Reflection: NONE

SKETCH GRAPH!



35. $f(x) = -|3x + 6| + 5$

$-|3(x+2)| + 5$

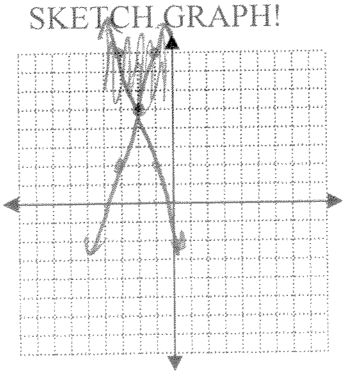
Name function: Abs. VALUE

Translation: LEFT 2, UP 5

Scale: HORIZONTAL SHRINK $\frac{1}{3}$

Reflection: About X-AXIS

SKETCH GRAPH!



36. $y = \sqrt{-x} + 3$

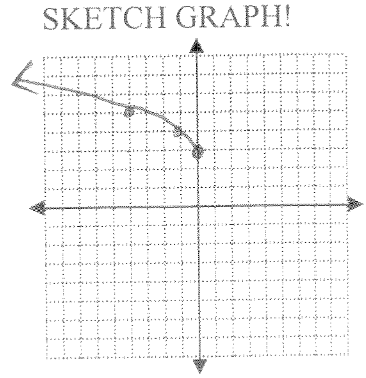
Name function: SQUARE ROOT

Translation: UP 3

Scale: NONE

Reflection: ABOUT Y-AXIS

SKETCH GRAPH!



Unit 5: Polynomials

Graph on your calculator to solve the following.

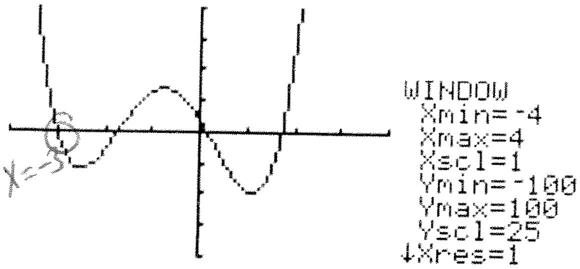
37. $0 = x^4 + 2x^2 - 3x - 1$

$x = -.279$ OR 1.157

38. $3x^6 - 2x^5 = 8 - 3x^2$

$3x^6 - 2x^5 + 3x^2 - 8 = 0$

39. Use the graph of the function to determine at least one zero, then find the exact values of all the zeros using the Factor Theorem. $f(x) = 7x^4 + 20x^3 - 24x^2 - 60x + 9$



$$\begin{array}{r|rrrrr} -3 & 7 & 20 & -24 & -60 & 9 \\ & & -21 & 3 & +63 & -9 \\ \hline & 7 & -1 & -21 & +3 & 0 \end{array}$$

$7x^3 - x^2 - 21x + 3$

$(7x^3 - x^2) + (-21x + 3)$

$x^2(7x - 1) + -3(7x - 1)$

$(x^2 - 3)(7x - 1)$

Factors: $(x+3)(x^2-3)(7x-1)$

$\boxed{\text{So: } x = -3, \pm\sqrt{3}, \frac{1}{7}}$

$x^2 - 3 = 0$, $x^2 = 3$, $x = \pm\sqrt{3}$

$7x - 1 = 0$, $7x = 1$, $x = \frac{1}{7}$

40. Factor the following.

a. $6x^2 + 13xy - 5y^2$

$x\ -30$, $+\ 13$

$(6x^2 - 3xy) + (10xy - 5y^2)$

$3x(2x - y) - 5y(2x - y)$

$\boxed{(3x - 5y)(2x - y)}$

b. $(5x^3 - 30x^2) + (8x + 48)$

$5x^2(x - 6) - 8(x - 6)$

$\boxed{(5x^2 - 8)(x - 6)}$

c. $216 + x^3$

SKIP

41. Factor to solve the following.

a. $4x^4 + 64x = 0$

$4x(x^3 + 16) = 0$

$4x = 0$

$\boxed{x = 0}$

$x^3 + 16 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{-16}$

$\boxed{x = -2\sqrt[3]{2}}$

b. $x^3 - 6x^2 + 8x = 0$

$x(x^2 - 6x + 8) = 0$

$x(x - 4)(x - 2) = 0$

$x = 0$, $x - 4 = 0$, $x - 2 = 0$

$x = 0$, $x = 4$, $x = 2$

c. $x^4 - 11x^2 = -30$

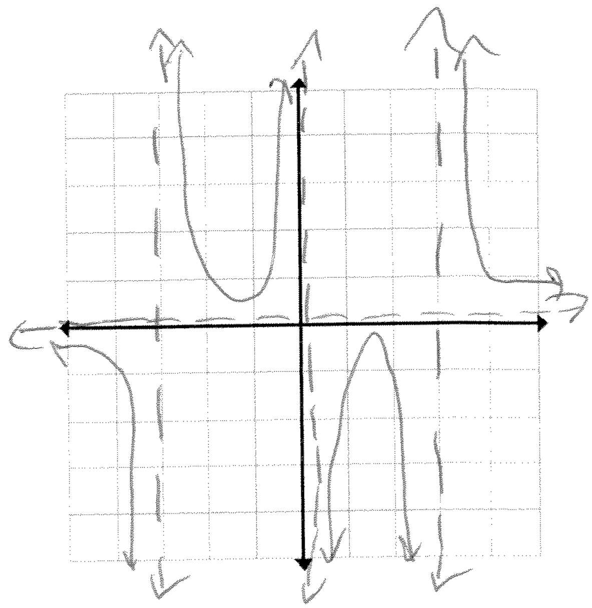
$x^4 - 11x^2 + 30 = 0$

$(x^2 - 6)(x^2 - 5) = 0$

$x^2 - 6 = 0$, $x^2 = 6$, $\boxed{x = \pm\sqrt{6}}$

$x^2 - 5 = 0$, $x^2 = 5$, $\boxed{x = \pm\sqrt{5}}$

Unit 6: Rational Functions



42. $f(x)=\dfrac{4}{x^3-9x}$ $\quad \dfrac{4}{x(x-3)(x+3)}$

Vertical Asymptotes/Holes:

$x=0$, $x=3$, $x=-3$ Vertical Asym

x-intercepts:

$y=0$

$0=\dfrac{4}{x^3-9x}$ NONE

$0=4$

y-intercepts: $x=0$

$\dfrac{4}{0^3-9(0)}=\dfrac{4}{0}$ None

Horizontal/Slant Asymptotes: $y=0$

Degree of TOP ~~B~~LESS THAN BOTTOM

Solve:

43. $\left[\dfrac{2x}{x+2}=\dfrac{5}{x^2-x-6}-\dfrac{1}{x-3}\right](x+2)(x-3)$ $\quad x\neq 3,-2$

$2x(x-3)=5-1(x+2)$

$2x^2-6x=5-x-2$

$2x^2-6x=3-x$

$2x^2-5x-3=0$

$(2x-6)(2x+1)=0$

$(x-3)(2x+1)=0$

$\cancel{x=3}$ or $2x+1=0$

$2x=-1$

$\boxed{x=\frac{-1}{2}}$

44. $\dfrac{7n^2-n}{n+9}=(2n-1)$ $\quad n\neq -9$

$7n^2-n=(2n-1)(n+9)$

$7n^2-n=2n^2+\cancel{18n-n}\,17n-9$

$5n^2-18n+9=0$

$(5n-15)(5n-3)=0$

$(n-3)(5n-3)=0$

$n-3=0$ $\quad 5n-3=0$

$n=3$ OR $n=3/5$

Simplify:

45. $\dfrac{\frac{3g}{g-5}+2}{\frac{3}{g-5}-4}$ $\dfrac{(g-5)}{(g-5)}$

$\dfrac{3g+2(g-5)}{3-4(g-5)}$

$\dfrac{3g+2g-10}{3-4g+20}=\boxed{\dfrac{5g-10}{-4g+23}}$

46. $\dfrac{x+4}{\sqrt{x}-\sqrt{x+2}}\cdot\dfrac{(\sqrt{x}+\sqrt{x+2})}{(\sqrt{x}+\sqrt{x+2})}$

$\dfrac{(x+4)(\sqrt{x}+\sqrt{x+2})}{x-(x+2)}$

$\boxed{\dfrac{(x+4)(\sqrt{x}+\sqrt{x+2})}{-2}}$

Unit 7: Exponential and Logarithmic Functions

Evaluate.

47. $\log_2 64 = x$

$2^x = 64$

$2^x = 2^6$

$\boxed{6}$

48. $\log_6 \frac{1}{36} = x$

$6^x = \frac{1}{36}$

$6^x = 6^{-2}$

$\boxed{-2}$

Solve for the indicated variable. Round to three digits where applicable.

49. $(5^{2x})^{(x+7)} = 1$

$5^{2x(x+7)} = 5^0$

$2x(x+7) = 0$

$2x = 0 \qquad x+7=0$

$\boxed{x=0} \qquad \boxed{x=-7}$

50. $56 \log_7 x = 203$

$\log_7 x = 3.625$

$7^{3.625} = x$

$\boxed{1157.4 = x}$

51. $e^{2x} - 22 = 25$

$\ln e^{2x} = \ln 47$

$2x = 3.85$

$\boxed{x = 1.925}$

52. ln x = ln (x + 14) – ln (x + 90)

$e^{\ln x} = e^{\ln \frac{(x+14)}{(x+90)}}$

$x = \frac{x+14}{x+90}$

$x(x+90) = x+14$

$x^2 + 90x = x + 14$

$x^2 + 9x + 14 = 0$

$(x+2)(x+7) = 0$

$\boxed{x=-2 \text{ or } x=-7}$

53. $5 \log_9(x-12) = 45$

$\log_9(x-12) = 9$

$9^9 = x - 12$

$9^9 + 12 = x$

$\boxed{387420501 = x}$

Find the inverse of the given function.

54. $f(x) = 8^{x-3}$

$\log x = \log 8^{y-3}$

$\log x = (y-3)\log 8$

$\frac{\log x}{\log 8} = y - 3$

$\boxed{\frac{\log x}{\log 8} + 3 = y}$ or $\boxed{1.107 \log x + 3 = y}$

55. ln y = 12 ln (x - 3)

$\ln x = 12 \ln(y-3)$

$e^{\ln x} = e^{\ln(y-3)^{12}}$

$x = (y-3)^{12}$

$\pm\sqrt[12]{x} = y - 3$

$\boxed{\pm\sqrt[12]{x} + 3 = y}$

56. At what rate compounded continuously will $12,000 have to be invested to amount to $35,000 in 12 years? Use A = Pe^{rt}.

$35{,}000 = 12{,}000\, e^{r \cdot 12}$

$\ln\left(\frac{35{,}000}{12{,}000}\right) = \ln e^{r \cdot 12}$

$\ln\left(\frac{35{,}000}{12{,}000}\right) = 12r$

$\frac{\ln\left(\frac{35{,}000}{12{,}000}\right)}{12} = r$

$\boxed{.089 = r}$

$\approx 8.9\%$

APPLICATIONS

The formula for the path of a flying bullet is given: $h = -9.8t^2 + vt + s$ where h = height of object, in meters t = time, in seconds, v = velocity, in meters per second and s = starting height, in meters

Bob shoots a gun straight up with a velocity of 300 meters per second and a starting height of 2 meters.
V S

57. What is the equation that represents this situation? $y =$ $-9.8t^2 + 300t + 2$

58. What do the axes represent?

x-axis: TIME in seconds

y-axis: height in meters

59. What does the y-intercept represent to Bob? Height when Bob shot

60. What do the x-intercepts represent to Bob? TIME WHEN BULLET HITS GROUND

61. How high is the bullet after 4 seconds? 1045.2

62. How long will it take for the bullet to hit the ground after it is fired? 30.62 sec

63. What is the maximum height of the bullet? $\frac{-b}{2a} = \frac{-300}{2(-9.8)} =$ $X = 15.3$ 2297.9 meters

64. At what time(s) will the bullet be 1200 meters in the air? 4.72 and 25.89 sec

65. The concentration C (in milligrams) of a certain drug in a patient's bloodstream t minutes after injection is given by $C(t) = \dfrac{50t}{(t^2 + 25)}$. Using your graphing calculator, graph $C(t)$ to answer the following questions.

a. What happens to the concentration of the drug as t increases?

IT DECREASES.

b. Determine the time at which the concentration is highest.

X = 5 minutes

c. What is the highest concentration?

5 milligrams

d. What is the horizontal asymptote of $C(t)$? What does it mean in this situation?

Y = 0 There will always be a slight amount of the drug in the blood stream.

e. If the drug is to be re-administered when the concentration decreases to 2 milligrams, how many minutes after the initial dose is the drug re-administered?

23.95 minutes