Unit 11 Review: Trig Identities

Directions for 1: Simplify to a single trig ratio or number.

1) $\frac{\tan x \csc x}{\sec x} = \frac{\sin}{\cos x}\cdot\left(\frac{1}{\sin x}\right)\left(\frac{1}{\sec x}\right)$

$= \frac{1}{\cancel{\cos x}}(\cancel{\cos x})$

$\boxed{=1}$

Directions for 2-4: Prove the identity. SHOW WORK!

2) $1 + sec^2\, x\, sin^2\, x = sec^2\, x$

$1+\frac{1}{\cos^2 x}\cdot \sin^2 x = \sec^2 x$

$1+\tan^2 x = \sec^2 x$

$\boxed{\sec^2 x = \sec^2 x}$

3) $\frac{\sec x-1}{\sec x+1}+\frac{\cos x-1}{\cos x+1}=0$

$\frac{(\sec x-1)(\cos x+1)+(\cos x-1)(\sec x+1)}{(\sec x+1)(\cos x+1)}=0$

$\frac{\sec x\cos x + \cancel{\sec x} - \cancel{\cos x} - 1 + \sec x\cos x + \cancel{\cos x} - \cancel{\sec x} - 1}{\cos x\sec x + \sec x + \cos x + 1}=0$

$\frac{2\sec x\cos x - 2}{\cos x\left(\frac{1}{\cos x}\right) + \sec x + \cos x + 1}=0$

$\frac{2\left(\frac{1}{\cos x}\right)(\cos x) - 2}{1 + \sec x + \cos x + 1}=0$

$\frac{2-2}{2+\sec x+\cos x}=0$

$\frac{0}{2+\sec x+\cos x}=0$

$0=0$

4) $\sin 2\alpha\,(\cot\alpha+\tan\alpha)=2$

$2\sin\alpha\cos\alpha\left(\frac{\cos\alpha}{\sin\alpha}+\frac{\sin\alpha}{\cos\alpha}\right)=2$

$2\cancel{\sin\alpha}\cos\alpha\left(\frac{\cos\alpha}{\cancel{\sin\alpha}}\right)+2\sin\alpha\cancel{\cos\alpha}\left(\frac{\sin\alpha}{\cancel{\cos\alpha}}\right)=2$

$2\cos^2\alpha + 2\sin^2\alpha = 2$

$2(\cos^2\alpha+\sin^2\alpha)=2$

$2(1)=2$

$2=2$ ✓

Directions for 5-6: Use the sum/difference/double or half angle formulas to find the exact value.

5) $\cos 22.5° = \cos\frac{45}{2}$

$\pm\sqrt{\frac{1+\cos x}{2}}$

$+\sqrt{\frac{1+\cos 45}{2}} = \sqrt{\frac{1+\frac{\sqrt2}{2}}{2}}$

$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt2}{2}}{2}}$

$= \sqrt{\frac{\frac{2+\sqrt2}{2}}{2}} = \boxed{\sqrt{\frac{2+\sqrt2}{4}}}$

6) $\sin \cancel{135}° \; 255° = \sin(210+45)$

$\sin 210\cos 45 + \sin 45\cos 210$

$-\frac{1}{2}\left(\frac{\sqrt2}{2}\right) + \frac{\sqrt2}{2}\left(\frac{-\sqrt3}{2}\right)$

$\frac{-\sqrt2}{4}+\frac{-\sqrt6}{4}$

$\boxed{\frac{-\sqrt2-\sqrt6}{4}}$

Directions for 7-8: If $\tan x = -\frac{40}{9}$ and x is in Quadrant II and $\sin y = -\frac{3}{5}$ and y is in Quadrant IV, find the exact value. Draw the reference triangle.

7) $\cos(x-y)$

$\cos x \cos y + \sin x \sin y$

$\left(\frac{-9}{41}\right)\left(\frac{4}{5}\right) + \left(\frac{40}{41}\right)\left(\frac{-3}{5}\right)$

$\frac{-36}{205} + -\frac{24}{41}$

$\boxed{\frac{-156}{205}}$

$\frac{90}{2} < \frac{x}{2} < \frac{180}{2}$

$45 < \frac{x}{2} < 90$

Positive

8) $\sin\left(\frac{x}{2}\right)$

$\pm\sqrt{\frac{1-\cos x}{2}} = \sqrt{\frac{1-\left(\frac{-9}{41}\right)}{2}}$

$= \sqrt{\frac{\frac{50}{41}}{2}}$

$= \sqrt{\frac{25}{41}} = \frac{\sqrt{25}}{\sqrt{41}}$

$= \frac{5}{\sqrt{41}} \cdot \frac{\sqrt{41}}{\sqrt{41}} = \boxed{\frac{5\sqrt{41}}{41}}$

Directions for 9: Find the exact value of the following where $0° \le x \le 360°$.

9) $tan^2 x - \sqrt{3}\tan x = 0$

$\tan x(\tan x - \sqrt{3}) = 0$

$\tan x = 0$ or $\tan x - \sqrt{3} = 0$

$\boxed{0, 360, 180}$

$\tan x = \sqrt{3}$

$\boxed{x = 60°, 240°}$

Directions for 10: Find the approximate value of the following where $0° \le x \le 360°$.

10) $10\sin x + 15 = 20 - 3\sin x$

$+3\sin x \quad -15 \quad -15 \quad +3\sin x$

$13\sin x = 5$

$\sin x = \frac{5}{13}$

$\boxed{x = 22.6° \text{ or } 157.4}$

22.6 22.6

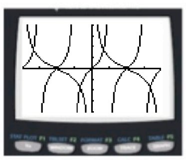
$180 - 22.6 = 157.4$

Applications/Extensions

1) For each equation use your calculator to determine if the following are identities. Sketch the graph (with zoom trig) to show they are or aren't identities. If the equation is an identity, VERIFY IT! If it is not an identity, find a value of x, $\left(x = 0, \pi, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}\right)$ for which both sides are defined but not equal.

a) $\frac{1+\cos x}{\sin x} = \tan x$

Sketch:



Verification or value of x it does not work for (SHOW WORK:

$x = 0 \rightarrow$ undefined because $\sin(0) = 0$

$x = \pi$: undefined because $\sin \pi = 0$

$x = \frac{\pi}{2}$: undefined because $\tan\frac{\pi}{2}$ is undefined

$\boxed{x = \frac{\pi}{4}}$: $\frac{1+\cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \tan\frac{\pi}{4}$

$\frac{1+\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\frac{\frac{2+\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$

$\left(\frac{2+\sqrt{2}}{2}\right)\left(\frac{2}{\sqrt{2}}\right) = 1$

$\frac{2+\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = 1$

$\frac{2\sqrt{2}+2}{2} = 1$

$\sqrt{2}+1 \neq 1$