AP Precalculus

Name: ______________________________ Date:________________ Period: _____

3B Review

Unit 3-B Review – Trigonometric Functions

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points. To be prepared, you must study all packets for lessons 3.8 – 3.15.

Write an equation that represents all asymptotes of the graph of f in the xy-plane.	**In the xy-plane, the angle θ is in standard position. What is the slope of the terminal ray of the angle?**
1. Let $f(\theta) = \tan\left(\frac{\theta}{6}\right)$. $\boldsymbol{\theta = 3\pi + k6\pi}$**, for integer values of** $\boldsymbol{k}$**.**	2. $\theta = \frac{7\pi}{4}$ $m = \dfrac{-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -1$

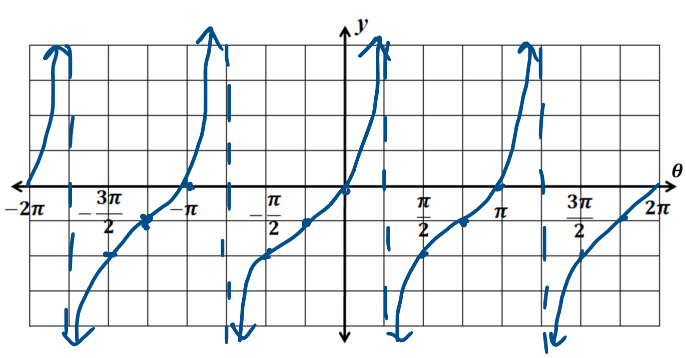
Evaluate.

3. $\tan\frac{\pi}{6}$ $\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \dfrac{1}{\sqrt{3}} = \dfrac{\sqrt{3}}{3}$	4. $\tan\frac{3\pi}{2}$ undefined	3. $\tan\frac{4\pi}{3}$ $\dfrac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = \sqrt{3}$

Graph each trig function.

6. $g(\theta) = \tan\left(\theta + \frac{\pi}{4}\right) - 1$

shift left $\frac{\pi}{4}$ and down 1

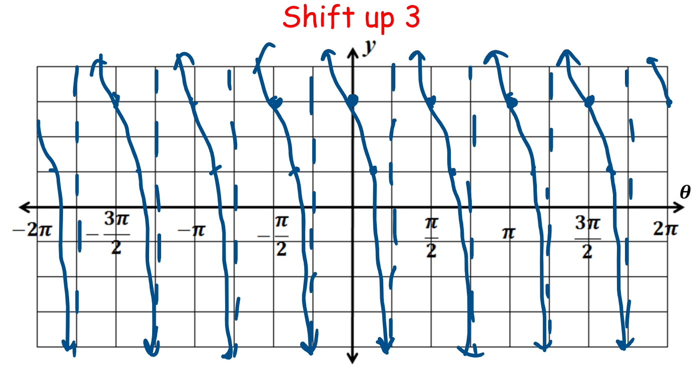


7. $g(\theta) = -2\tan(2\theta) + 3$

vertical reflection, vertical dilation of 2

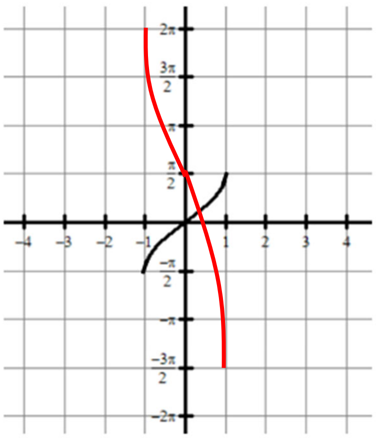
Horizontal dilation of $\frac{1}{2}$

Shift up 3

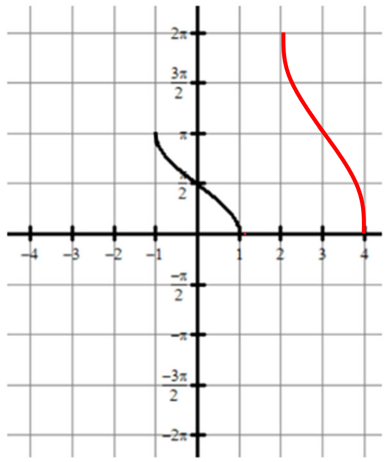


The parent function is shown below. Use the parent function to graph $g(x)$.

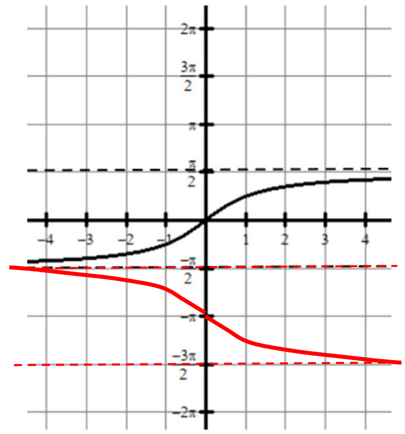
8. $g(x) = 3\sin^{-1}(-x) + \frac{\pi}{2}$



9. $g(x) = 2\cos^{-1}(x-3)$



10. $g(x) = -\tan^{-1}(x) - \pi$



Find the inverse of each function and list the domain and range of $f^{-1}(x)$.

11. $f(x) = 2\sin x - 5$ for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$x = 2\sin y - 5$

$+5 \qquad +5$

$\frac{x+5}{2} = \frac{2\sin y}{2}$

$\frac{x+5}{2} = \sin y$

$\sin^{-1}\left(\frac{x+5}{2}\right) = \sin^{-1}(\sin y)$

$\boxed{\sin^{-1}\left(\frac{x+5}{2}\right) = y}$

domain: $-7 \le x \le -3$ ← $2\sin\left(-\frac{\pi}{2}\right) - 5$, $2(-1) - 5$, -7

range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

$2\sin\left(\frac{\pi}{2}\right) - 5$, $2(1) - 5$, -3

Solve each equation for $0 \le \theta \le 2\pi$. Find the exact value(s) using the unit circle

12. $2\sin\theta + 5 = 4$

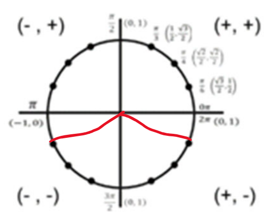
$-5 \qquad -5$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

$\theta = \sin^{-1}\left(-\frac{1}{2}\right)$

$\boxed{\theta = \frac{7\pi}{6} \text{ and } \frac{11\pi}{6}}$



13. $\cos^2\theta + \cos\theta = 0$

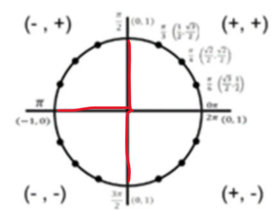
$\cos\theta(\cos\theta + 1) = 0$

$\cos\theta = 0$ | $\cos\theta + 1 = 0$

$\theta = \cos^{-1}(0)$ | $\cos\theta = -1$

$\theta = \cos^{-1}(-1)$

$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \pi}$



Solve each equation. Find ALL approximate value(s) using a calculator.

14. $5 + 5\sin^2 x = 8$

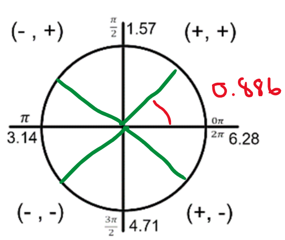
$-5 \qquad -5$

$\frac{5\sin^2 x}{5} = \frac{3}{5}$

$\sqrt{\sin^2 x} = \sqrt{\frac{3}{5}}$

$\sin x = \pm 0.774$

$\boxed{x = 0.886 + \pi n \text{ and } 2.255 + \pi n}$



15. $\tan^2 x - 3\tan x = 18$

$-18 \qquad -18$

$\tan^2 x - 3\tan x - 18 = 0$

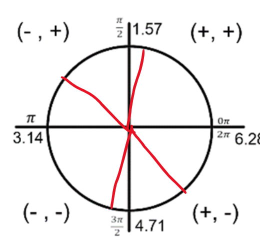
$(\tan x - 6)(\tan x + 3)$

$\tan x - 6 = 0$ | $\tan x + 3 = 0$

$\tan x = 6$ | $\tan x = -3$

$x = \tan^{-1}(6)$ | $x = \tan^{-1}(-3)$

$\boxed{x = 1.405 + \pi n \qquad x = -1.249 + \pi n}$



Evaluate the following expressions. Use exact values.

16. $\sec\left(\frac{\pi}{3}\right)$

$\frac{1}{\cos\frac{\pi}{3}} = \frac{1}{\frac{1}{2}} = \boxed{2}$

17. $\csc\left(-\frac{\pi}{2}\right)$

$\frac{1}{\sin\left(-\frac{\pi}{2}\right)} = \frac{1}{-1} = \boxed{-1}$

18. $\cot\left(\frac{5\pi}{6}\right)$

$\frac{\cos\left(\frac{5\pi}{6}\right)}{\sin\left(\frac{5\pi}{6}\right)} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{-\sqrt{3}}$

Evaluate the following expressions. Use approximate values from calculator.

19. $\csc(1.84)$

$\frac{1}{\sin(1.84} = 1.037$

20. $\sec\left(\frac{3\pi}{5}\right)$

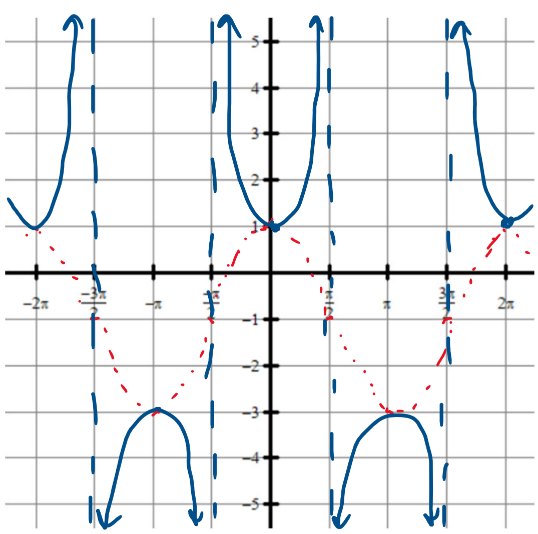
$\frac{1}{\cos\left(\frac{3\pi}{5}\right)} = -3.236$

21. $\cot\left(\frac{4\pi}{9}\right)$

$\frac{1}{\tan\left(\frac{4\pi}{9}\right)} = 0.176$

Graph the following. State the range and all vertical asymptotes.

22. $f(x) = 2\csc\left(x + \frac{\pi}{2}\right) - 1$



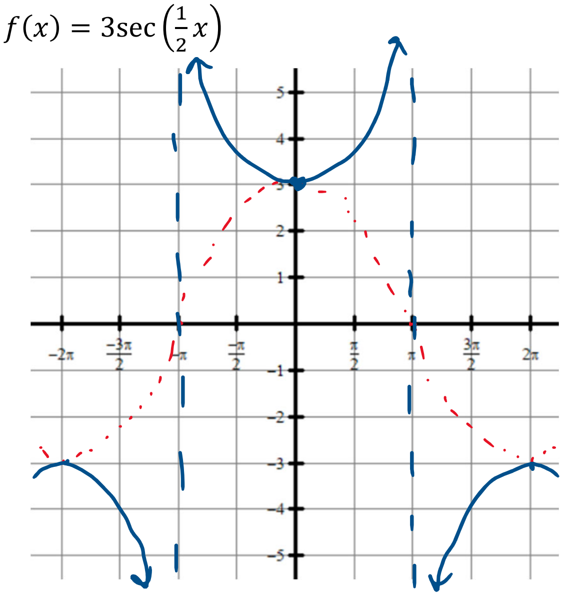
Range:

$(-\infty, -3] \cup [1, \infty)$

Vertical Asymptotes:

$x = \frac{\pi}{2} + \pi n$

23. $f(x) = 3\sec\left(\frac{1}{2}x\right)$



Range:

$(-\infty, -3] \cup [3, \infty)$

Vertical Asymptotes:

$x = \pi + 2\pi n$

Use trig identities to write each expression in terms of a single trig identity.

24. $\cos^2 x \sec x$

$\cos^2 x \cdot \frac{1}{\cos x}$

$\frac{\cos^2 x}{\cos x}$

$\boxed{\cos x}$

25. $(1 - \sin^2 x) \csc^2 x$

$\cos^2 x \cdot \frac{1}{\sin^2 x}$

$\frac{\cos^2 x}{\sin^2 x}$

$\boxed{\cot^2 x}$

26. $\frac{1}{1 + \cot^2 x} = \frac{1}{\csc^2 x}$

$\frac{1}{\frac{1}{\sin^2 x}}$

$\boxed{\sin^2 x}$

Find the exact value of the sum or difference.

27. $\sin\left(\frac{3\pi}{4} - \frac{5\pi}{6}\right)$

$\sin \frac{3\pi}{4} \cos \frac{5\pi}{6} - \sin \frac{5\pi}{6} \cos \frac{3\pi}{4}$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$-\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$\boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}$

28. $\cos\left(\frac{\pi}{2} + \frac{2\pi}{3}\right)$

$\cos \frac{\pi}{2} \cos \frac{2\pi}{3} - \sin \frac{\pi}{2} \sin \frac{2\pi}{3}$

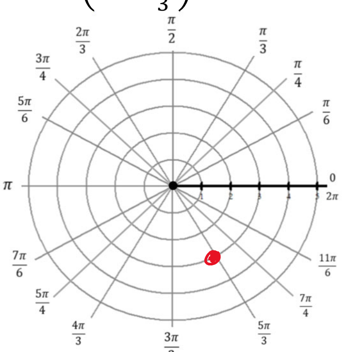
$(0)\left(-\frac{1}{2}\right) - (1)\left(\frac{\sqrt{3}}{2}\right)$

$0 - \frac{\sqrt{3}}{2}$

$\boxed{-\frac{\sqrt{3}}{2}}$

Polar Coordinates

29. Plot $\left(-3, \frac{2\pi}{3}\right)$



30. Convert from polar to rectangular.

$\left(4, \frac{5\pi}{6}\right)$

$x = 4\cos \frac{5\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right)$

$y = 4 \sin \frac{5\pi}{6} = 4\left(\frac{1}{2}\right)$

$(-2\sqrt{3}, 2)$

31. Convert from rectangular to polar where $0 \le \theta \le 2\pi$.

$(2, -4)$

$r = \sqrt{(2)^2 + (-4)^2} = \sqrt{20}$

$\theta = \tan^{-1}\left(-\frac{4}{2}\right) = -1.107 = 5.176$

$(\sqrt{20}, 5.176)$

32. Convert from rectangular complex to polar.

$-3 - 5i$

$r = \sqrt{(-3)^2 + (-5)^2} = \sqrt{34}$

$\theta = \tan^{-1}\left(\frac{-5}{-3}\right) = 1.03$

$\sqrt{34}\left[\cos(1.03) + i \sin(1.03)\right]$

33. Convert from polar complex to rectangular.

$6\left[\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right)\right]$

$x = 6\cos\left(\frac{3\pi}{4}\right) = 6\left(-\frac{\sqrt{2}}{2}\right) = -3\sqrt{2}$

$y = 6\sin\left(\frac{3\pi}{4}\right) = 6\left(\frac{\sqrt{2}}{2}\right) = 3\sqrt{2}$

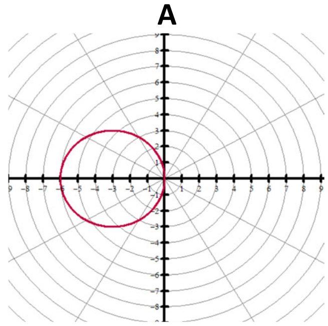
$-3\sqrt{2} + 3i\sqrt{2}$

Match the equation to its graph below.

34. $r = 3 + 6\cos(\theta)$ matches graph: C	35. $r = 3 - 6\sin(\theta)$ matches graph: E	36. $r = 3 - 3\cos(\theta)$ matches graph: I
37. $r = 6 - 3\cos(\theta)$ matches graph: F	38. $r = -6\cos(\theta)$ matches graph: A	39. $r = 6\sin(3\theta)$ matches graph: L
40. $r = 3\cos(6\theta)$ matches graph: J	41. $r = \frac{\theta}{3}$ matches graph: H	42. $r = -6\cos(3\theta)$ matches graph: B
43. $r = 3\sin(6\theta)$ matches graph: G	44. $r = 6$ matches graph: D	45. $\theta = \frac{\pi}{3}$ matches graph: K

GRAPHS:

A



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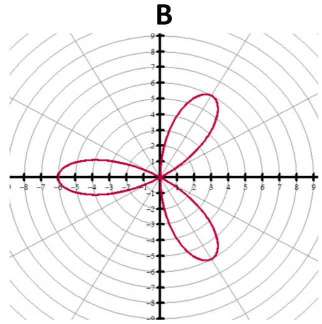
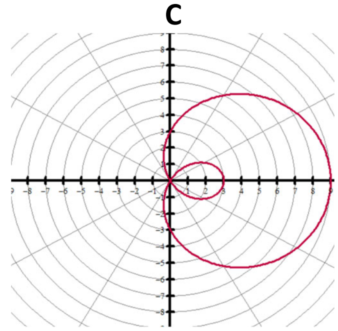


C



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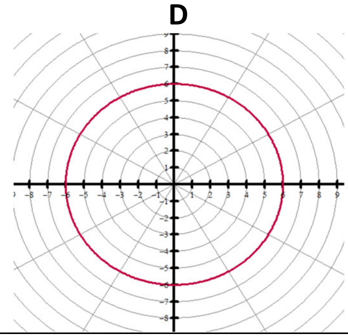
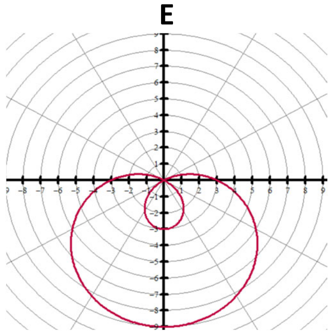


E



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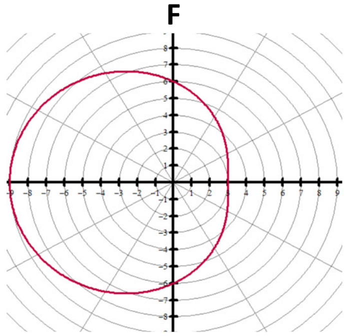
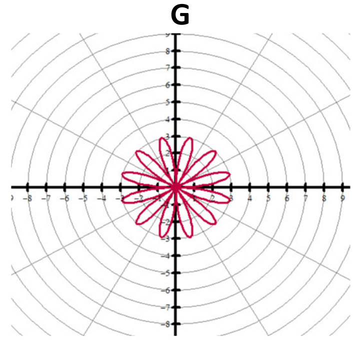
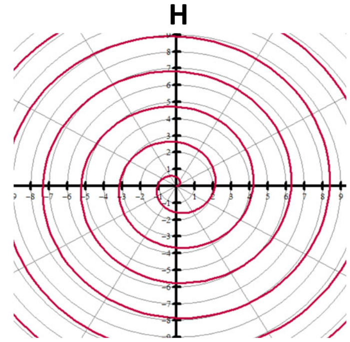


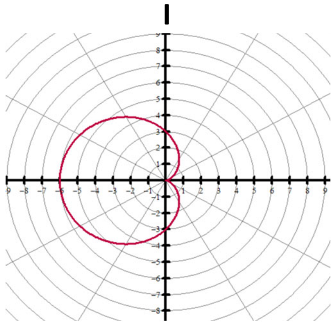
G



H



I



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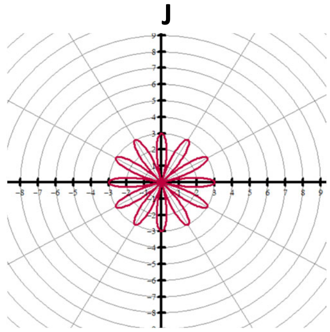

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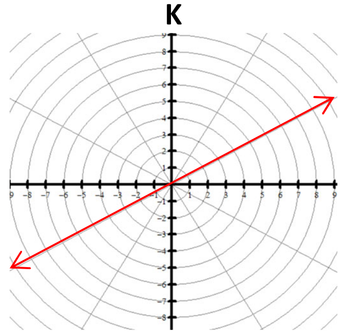
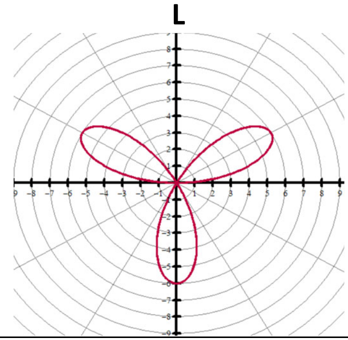


L



Use the table of selected values for the polar function $r = f(\theta)$ to answer the following.

46.

a. Is f increasing or decreasing on the interval $0 \le \theta \le \frac{\pi}{2}$?

θ	r
0	-5
$\frac{\pi}{8}$	-4.619
$\frac{\pi}{4}$	-3.536
$\frac{3\pi}{8}$	-1.913
$\frac{\pi}{2}$	0

b. Is the distance between $f(\theta)$ and the pole is increasing or decreasing on the interval $0 \le \theta \le \frac{\pi}{2}$?

r is negative
increasing

c. Find the average rate of change of f between $\theta = \frac{\pi}{8}$ and $\theta = \frac{3\pi}{8}$.

$$\frac{-4.619-(-1.913)}{\frac{\pi}{8}-\frac{3\pi}{8}} = \frac{-2.07}{-\frac{2\pi}{8}} = \frac{-2.07}{-\frac{\pi}{4}} = 3.445$$

d. Estimate the value of $f\left(\frac{\pi}{4}\right)$ using an average rate of change.

$y - y_1 = m(x - x_1)$

$y - -4.619 = 3.445\left(x - \frac{\pi}{8}\right)$

$y + 4.619 = 3.445x - 1.353$

$\quad -4.619 \qquad\qquad -4.619$

$y = 3.445\left(\frac{\pi}{4}\right) - 5.972$

$y = -3.266$

$f\left(\frac{\pi}{4}\right) \approx -3.266$

Multiple Choice

47. The function f is given by $f(x) = 4\sin(x) + 1$. For what values of x where $0 \le x \le 2\pi$ is $f(x) \le -1$?

(A) $\frac{7\pi}{6} \le x \le \frac{11\pi}{6}$

(B) $\frac{4\pi}{3} \le x \le \frac{5\pi}{3}$

(C) $0 \le x \le \frac{7\pi}{6}$ and $\frac{11\pi}{6} \le x \le 2\pi$

(D) $0 \le x \le \frac{4\pi}{3}$ and $\frac{5\pi}{3} \le x \le 2\pi$

$4\sin x + 1 \le -1$

$\quad -1 \qquad -1$

$\frac{4\sin x}{4} \le \frac{-2}{4}$

$\sin x \le -\frac{1}{2}$

$x = \sin^{-1}\left(-\frac{1}{2}\right)$

$x = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$